TALK DÜSSELDORF

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1. Local Action. Let $T_d = (V, E)$ denote the d-regular tree ($d \in \mathbb{N}_{\geq 3}$) and let $H \leq \mathrm{Aut}(T_d)$. Given $x \in V$, the *local action* of H at x is the permutation group

$$H_x \curvearrowright E(x) := \{e \in E \mid x \in e\}.$$

Let Ω be a set. A permutation group $F \leq \mathrm{Sym}(\Omega)$ could be

2-transitive	$\Rightarrow$	primitive	$\Rightarrow$	quasiprimitive	$\Rightarrow$	semiprimitive	$\Rightarrow$	transitive
		A_3		$A_5 \curvearrowright A_5/C_5$		$C_4 \trianglerighteq C_2$		$D_4 \trianglerighteq C_2 \times C_2$

2. Burger–Mozes Theory. The fundamental definitions of Burger–Mozes theory are meaningful in the setting of totally disconnected locally compact groups: Let H be a t.d.l.c. group. We define

$$H^{(\infty)} := \bigcap \{N \leq H \mid N \text{ closed normal cocompact}\},$$
$$\mathrm{QZ}(H) := \{h \in H \mid C_H(h) \leq H \text{ is open}\}.$$

Theorem 1 (Burger–Mozes '00). Let $H \leq \mathrm{Aut}(T_d)$ be closed, non-discrete and locally quasiprimitive.

(i) $H^{(\infty)}$ is minimal closed normal cocompact in H.
(ii) $\mathrm{QZ}(H)$ is maximal discrete normal, and non-cocompact in H.
(iii) $H^{(\infty)}/\mathrm{QZ}(H^{(\infty)}) = H^{(\infty)}/(\mathrm{QZ}(H) \cap H^{(\infty)})$ admits minimal, non-trivial closed normal subgroups; finite in number, H-conjugate and topologically simple.

If, in addition, H is locally primitive then

(iv) $H^{(\infty)}/\mathrm{QZ}(H^{(\infty)})$ is a direct product of topologically simple groups.

3. Groups Acting on Trees with Non-Trivial Quasi-Center.

Theorem 2 (Burger–Mozes '00, T. '17). Let $H \leq \mathrm{Aut}(T_d)$ be non-discrete. If H is locally

(i) transitive then $\mathrm{QZ}(H)$ contains no inversion.
(ii) semiprimitive then $\mathrm{QZ}(H)$ contains no non-trivial edge-fixating element.
(iii) quasiprimitive then $\mathrm{QZ}(H)$ contains no non-trivial elliptic element.
(iv) k-transitive ($k \in \mathbb{N}$) then $\mathrm{QZ}(H)$ contains no hyperbolic element of length k.

Theorem 3 (T. '17). There is a closed, non-discrete, compactly generated subgroup of $\mathrm{Aut}(T_d)$ which is locally

(i) intransitive and contains a quasi-central inversion.
(ii) transitive and contains a non-trivial quasi-central edge-fixating element.
(iii) semiprimitive and contains a non-trivial quasi-central elliptic element.
(iv) (a) intransitive and contains a quasi-central hyperbolic element of length 1.
(b) quasiprimitive and contains a quasi-central hyperbolic element of length 2.

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