One question to ask yourself about everything you do

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Why aren't you studying groups acting on trees?

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TED

Technology

TED

Technology Entertainment

TED

Technology Entertainment Design

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Technology	Design

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totally

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Design

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totally disconnected

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  There exist continuously many different varieties of groups.
  (closed under homomorphic images, subgroups, cartesian products)

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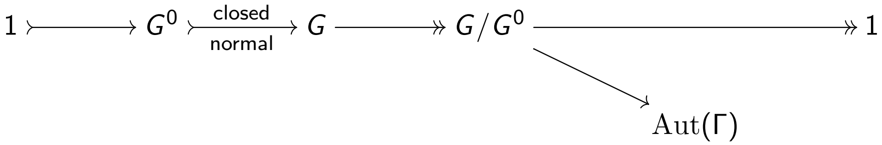

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$$
\begin{array}{ccccccccc}
1 & \rightarrowtail & G^0 & \overset{\text{closed}}{\underset{\text{normal}}{\rightarrowtail}} & G & \twoheadrightarrow & G/G^0 & \twoheadrightarrow & 1 \\
 & & & & & & & \searrow & \\
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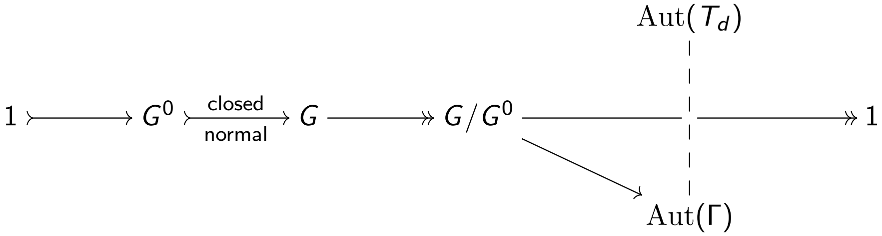

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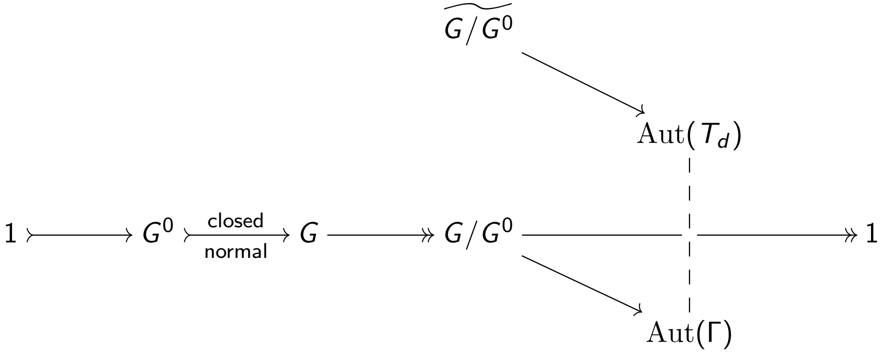

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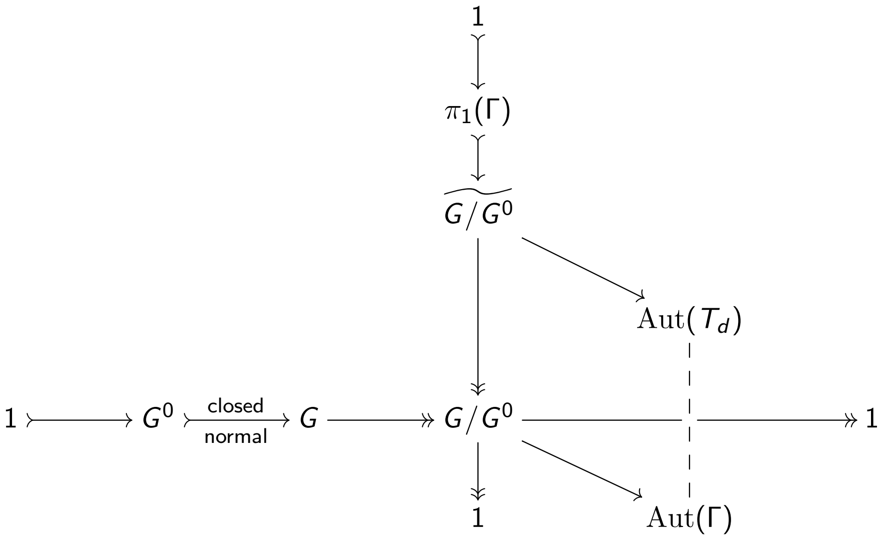

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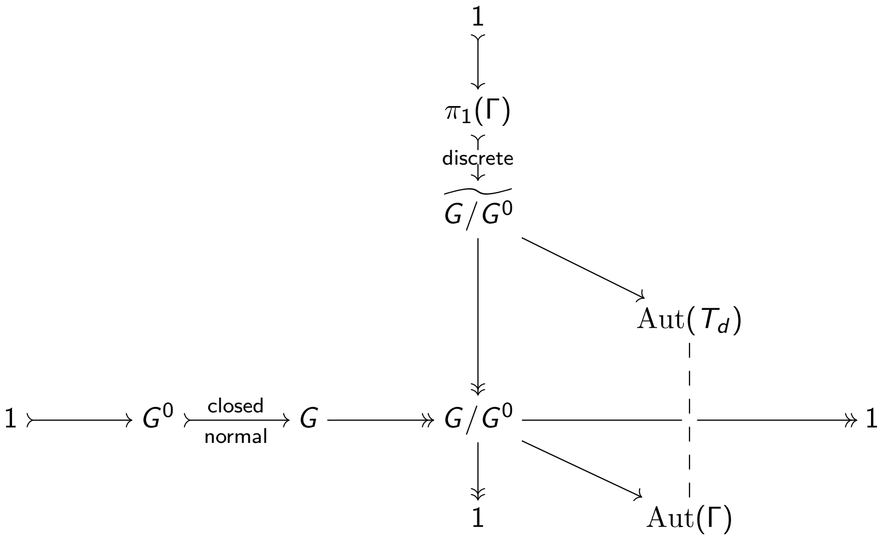

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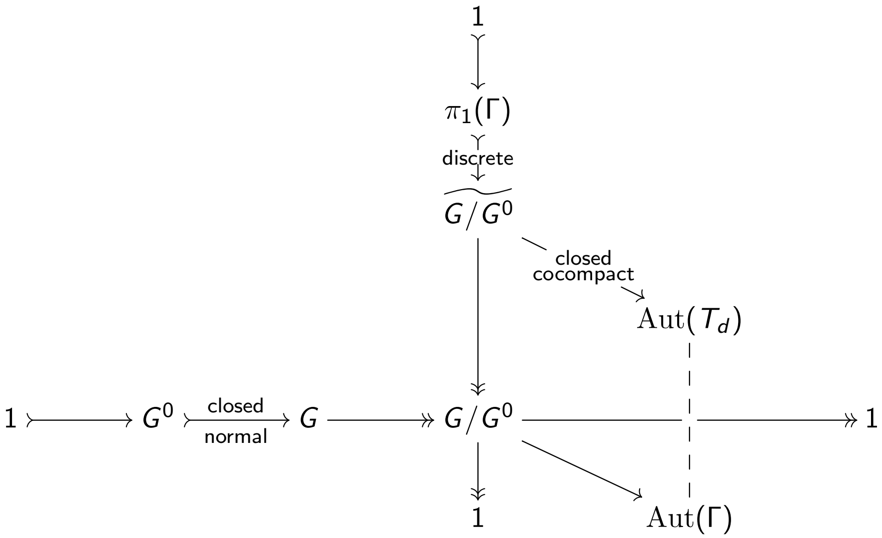

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