Think Globally, Act Locally

Stephan Tornier

THE UNIVERSITY OF NEWCASTLE AUSTRALIA

September 12, 2019

Let G be a group.

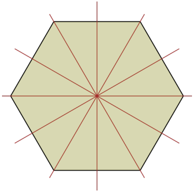
Let G be a group. For example...

Let G be a group. For example...

Group Theory

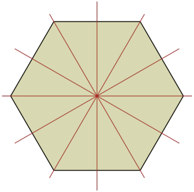
Let G be a group. For example...

Group Theory



Let G be a group. For example...

Group Theory

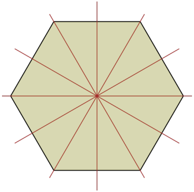


D_6

Let G be a group. For example...

Group Theory

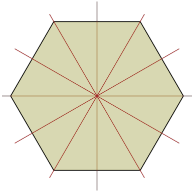
Linear Algebra



D_6

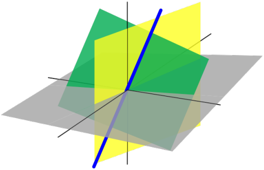
Let G be a group. For example...

Group Theory



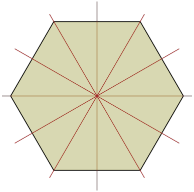
D_6

Linear Algebra



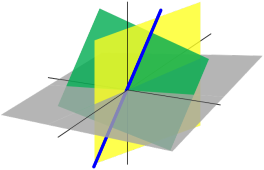
Let G be a group. For example...

Group Theory



D_6

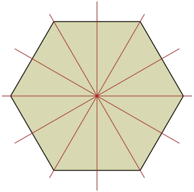
Linear Algebra



$\mathrm{GL}(n, \mathbb{R})$

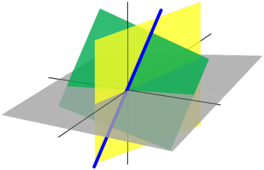
Let G be a group. For example...

Group Theory



D_6

Linear Algebra

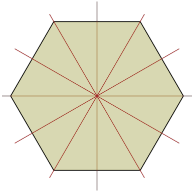


$\mathrm{GL}(\mathrm{n}, \mathbb{R})$

Number Theory

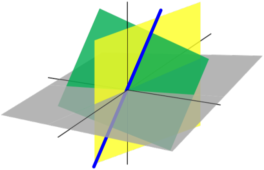
Let G be a group. For example...

Group Theory



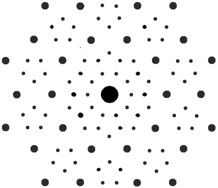
D_6

Linear Algebra



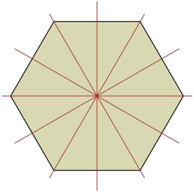
$\mathrm{GL}(\mathrm{n}, \mathbb{R})$

Number Theory



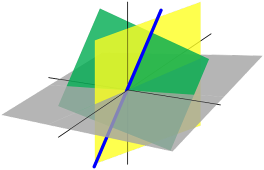
Let G be a group. For example...

Group Theory



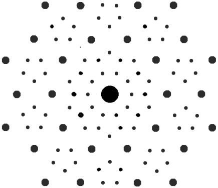
D_6

Linear Algebra



$\mathrm{GL}(\mathrm{n}, \mathbb{R})$

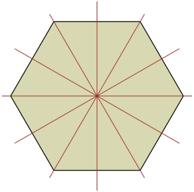
Number Theory



D_{10}

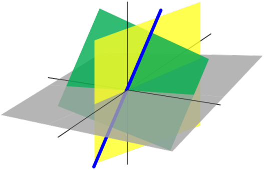
Let G be a group. For example...

Group Theory



D_6

Linear Algebra

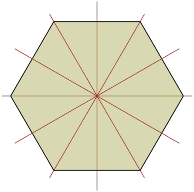


$\mathrm{GL}(n, \mathbb{R})$

Number Theory

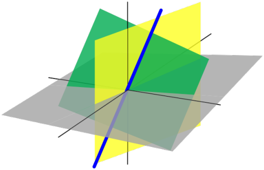
Let G be a group. For example...

Group Theory



D_6

Linear Algebra



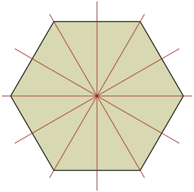
$\mathrm{GL}(\mathrm{n}, \mathbb{R})$

Number Theory

$K \subseteq E$ fields

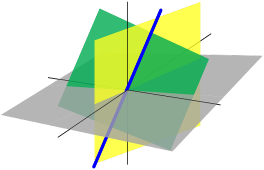
Let G be a group. For example...

Group Theory



D_6

Linear Algebra



$\mathrm{GL}(\mathrm{n}, \mathbb{R})$

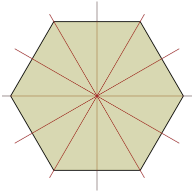
Number Theory

$K \subseteq E$ fields

$\mathbb{Q} \subseteq \mathbb{Q}(\sqrt{2}, \sqrt{3})$

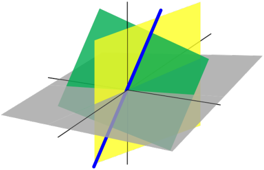
Let G be a group. For example...

Group Theory



D_6

Linear Algebra



$\mathrm{GL}(\mathrm{n}, \mathbb{R})$

Number Theory

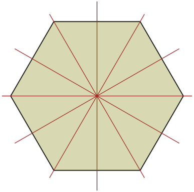
$K \subseteq E$ fields

$\mathbb{Q} \subseteq \mathbb{Q}(\sqrt{2}, \sqrt{3})$

$\mathbb{F}_p \subseteq \overline{\mathbb{F}_p(X)}$

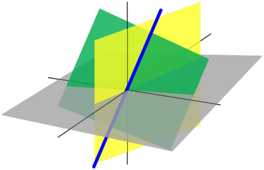
Let G be a group. For example...

Group Theory



D_6

Linear Algebra



$\mathrm{GL}(\mathrm{n}, \mathbb{R})$

Number Theory

$K \subseteq E$ fields

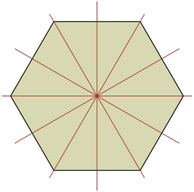
$\mathbb{Q} \subseteq \mathbb{Q}(\sqrt{2}, \sqrt{3})$

$\mathbb{F}_p \subseteq \overline{\mathbb{F}_p(X)}$

$\mathrm{Aut}(E/K)$

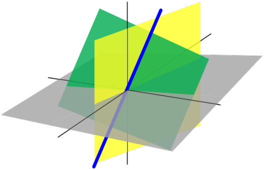
Let G be a group. For example...

Group Theory



D_6

Linear Algebra



$\mathrm{GL}(\mathrm{n}, \mathbb{R})$

Number Theory

$K \subseteq E$ fields

$\mathbb{Q} \subseteq \mathbb{Q}(\sqrt{2}, \sqrt{3})$

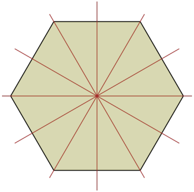
$\mathbb{F}_p \subseteq \overline{\mathbb{F}_p(X)}$

$\mathrm{Aut}(E/K)$

Differential Equations

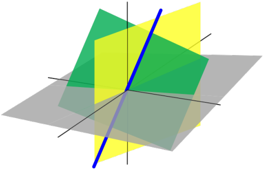
Let G be a group. For example...

Group Theory



D_6

Linear Algebra



$\mathrm{GL}(\mathrm{n}, \mathbb{R})$

Number Theory

$K \subseteq E$ fields

$\mathbb{Q} \subseteq \mathbb{Q}(\sqrt{2}, \sqrt{3})$

$\mathbb{F}_p \subseteq \overline{\mathbb{F}_p(X)}$

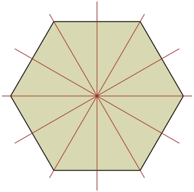
$\mathrm{Aut}(E/K)$

Differential Equations

$\Delta f = 0$

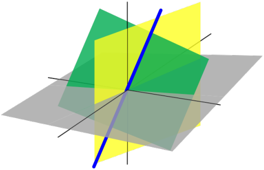
Let G be a group. For example...

Group Theory



D_6

Linear Algebra



$\mathrm{GL}(\mathrm{n}, \mathbb{R})$

Number Theory

$K \subseteq E$ fields

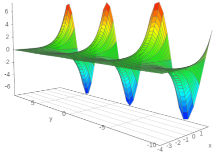
$\mathbb{Q} \subseteq \mathbb{Q}(\sqrt{2}, \sqrt{3})$

$\mathbb{F}_p \subseteq \overline{\mathbb{F}_p(X)}$

$\mathrm{Aut}(E/K)$

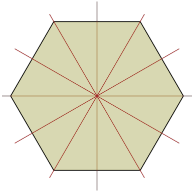
Differential Equations

$\Delta f = 0$



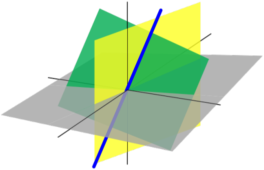
Let G be a group. For example...

Group Theory



D_6

Linear Algebra



$\mathrm{GL}(\mathrm{n}, \mathbb{R})$

Number Theory

$K \subseteq E$ fields

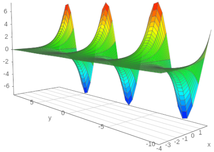
$\mathbb{Q} \subseteq \mathbb{Q}(\sqrt{2}, \sqrt{3})$

$\mathbb{F}_p \subseteq \overline{\mathbb{F}_p(X)}$

$\mathrm{Aut}(E/K)$

Differential Equations

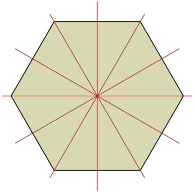
$\Delta f = 0$



$\mathrm{O}(n)$

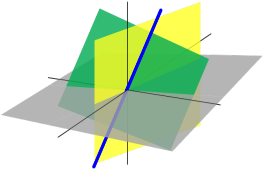
Let G be a group. For example...

Group Theory



D_6

Linear Algebra



$\mathrm{GL}(\mathrm{n}, \mathbb{R})$

Number Theory

$K \subseteq E$ fields

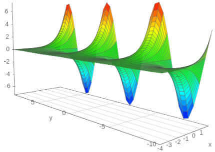
$\mathbb{Q} \subseteq \mathbb{Q}(\sqrt{2}, \sqrt{3})$

$\mathbb{F}_p \subseteq \overline{\mathbb{F}_p(X)}$

$\mathrm{Aut}(E/K)$

Differential Equations

$\Delta f = 0$

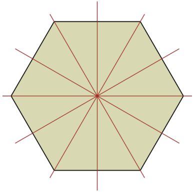


$\mathrm{O}(n)$

Differential Geometry

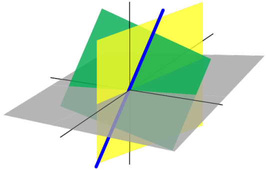
Let G be a group. For example...

Group Theory



D_6

Linear Algebra



$\mathrm{GL}(\mathrm{n}, \mathbb{R})$

Number Theory

$K \subseteq E$ fields

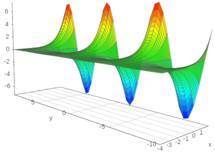
$\mathbb{Q} \subseteq \mathbb{Q}(\sqrt{2}, \sqrt{3})$

$\mathbb{F}_p \subseteq \overline{\mathbb{F}_p(X)}$

$\mathrm{Aut}(E/K)$

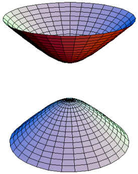
Differential Equations

$\Delta f = 0$



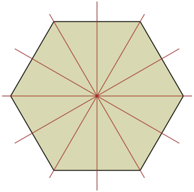
$\mathrm{O}(n)$

Differential Geometry



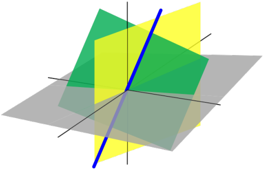
Let G be a group. For example...

Group Theory



D_6

Linear Algebra



$\mathrm{GL}(\mathrm{n}, \mathbb{R})$

Number Theory

$K \subseteq E$ fields

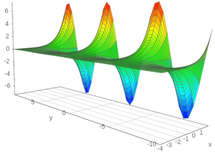
$\mathbb{Q} \subseteq \mathbb{Q}(\sqrt{2}, \sqrt{3})$

$\mathbb{F}_p \subseteq \overline{\mathbb{F}_p(X)}$

$\mathrm{Aut}(E/K)$

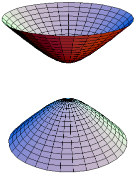
Differential Equations

$\Delta f = 0$



$\mathrm{O}(n)$

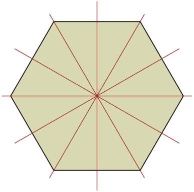
Differential Geometry



$\mathrm{O}(1, n)$

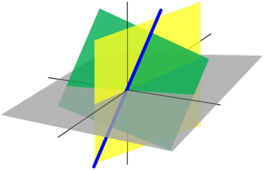
Let G be a group. For example...

Group Theory



D_6

Linear Algebra



$\mathrm{GL}(\mathrm{n}, \mathbb{R})$

Number Theory

$K \subseteq E$ fields

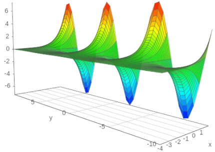
$\mathbb{Q} \subseteq \mathbb{Q}(\sqrt{2}, \sqrt{3})$

$\mathbb{F}_p \subseteq \overline{\mathbb{F}_p(X)}$

$\mathrm{Aut}(E/K)$

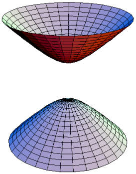
Differential Equations

$\Delta f = 0$



$\mathrm{O}(n)$

Differential Geometry

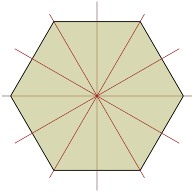


$\mathrm{O}(1, n)$

Graph Theory

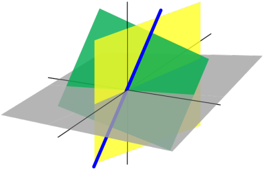
Let G be a group. For example...

Group Theory



D_6

Linear Algebra



$\mathrm{GL}(\mathrm{n}, \mathbb{R})$

Number Theory

$K \subseteq E$ fields

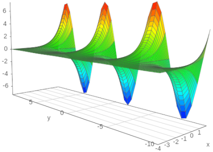
$\mathbb{Q} \subseteq \mathbb{Q}(\sqrt{2}, \sqrt{3})$

$\mathbb{F}_p \subseteq \overline{\mathbb{F}_p(X)}$

$\mathrm{Aut}(E/K)$

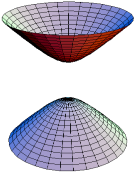
Differential Equations

$\Delta f = 0$



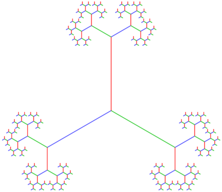
$\mathrm{O}(n)$

Differential Geometry



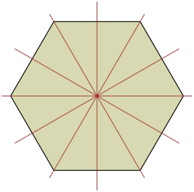
$\mathrm{O}(1, n)$

Graph Theory



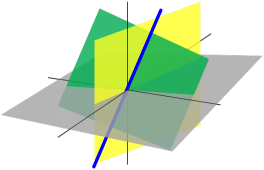
Let G be a group. For example...

Group Theory



D_6

Linear Algebra



$\mathrm{GL}(\mathrm{n}, \mathbb{R})$

Number Theory

$K \subseteq E$ fields

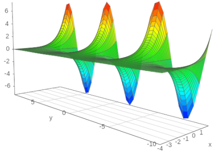
$\mathbb{Q} \subseteq \mathbb{Q}(\sqrt{2}, \sqrt{3})$

$\mathbb{F}_p \subseteq \overline{\mathbb{F}_p(X)}$

$\mathrm{Aut}(E/K)$

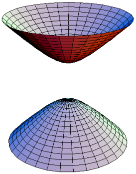
Differential Equations

$\Delta f = 0$



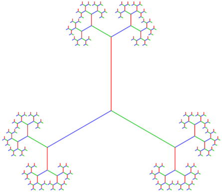
$\mathrm{O}(n)$

Differential Geometry



$\mathrm{O}(1, n)$

Graph Theory



$\mathrm{Aut}(T_d)$

Let G be a group.

Let G be a group.

Finite

Let G be a group.

Finite

- Composition series:

Let G be a group.

Finite

- Composition series:
  $1 = G_0 \trianglelefteq G_1 \trianglelefteq \cdots \trianglelefteq G_{n-1} \trianglelefteq G_n = G$

Let G be a group.

Finite

- Composition series:
  $1 = G_0 \trianglelefteq G_1 \trianglelefteq \cdots \trianglelefteq G_{n-1} \trianglelefteq G_n = G$ where G_{i+1}/G_i is simple.

Let G be a group.

Finite

- Composition series:
  $1 = G_0 \trianglelefteq G_1 \trianglelefteq \cdots \trianglelefteq G_{n-1} \trianglelefteq G_n = G$ where G_{i+1}/G_i is simple.
- Jordan-Hölder: Uniqueness of subquotients.

Let G be a group.

Finite

- Composition series:
  $1 = G_0 \trianglelefteq G_1 \trianglelefteq \cdots \trianglelefteq G_{n-1} \trianglelefteq G_n = G$ where G_{i+1}/G_i is simple.
- Jordan-Hölder: Uniqueness of subquotients.
- Classification of finite simple groups.

Let G be a group.

Finite

- Composition series:
  $1 = G_0 \trianglelefteq G_1 \trianglelefteq \cdots \trianglelefteq G_{n-1} \trianglelefteq G_n = G$ where G_{i+1}/G_i is simple.
- Jordan-Hölder: Uniqueness of subquotients.
- Classification of finite simple groups.

Infinite

Let G be a group.

Finite

- Composition series:
  $1 = G_0 \trianglelefteq G_1 \trianglelefteq \cdots \trianglelefteq G_{n-1} \trianglelefteq G_n = G$ where G_{i+1}/G_i is simple.
- Jordan-Hölder: Uniqueness of subquotients.
- Classification of finite simple groups.

Infinite

- Adian-Rabin '55:

Let G be a group.

Finite

- Composition series:
  $1 = G_0 \trianglelefteq G_1 \trianglelefteq \cdots \trianglelefteq G_{n-1} \trianglelefteq G_n = G$ where G_{i+1}/G_i is simple.
- Jordan-Hölder: Uniqueness of subquotients.
- Classification of finite simple groups.

Infinite

- Adian-Rabin '55:
  The isomorphism problem for finitely presented groups is undecidable.

Let G be a group.

Finite

- Composition series:
  $1 = G_0 \trianglelefteq G_1 \trianglelefteq \cdots \trianglelefteq G_{n-1} \trianglelefteq G_n = G$ where G_{i+1}/G_i is simple.
- Jordan-Hölder: Uniqueness of subquotients.
- Classification of finite simple groups.

Infinite

- Adian-Rabin '55:
  The isomorphism problem for finitely presented groups is undecidable.
- Olshansky, Vaughan-Lee '70:

Let G be a group.

Finite

- Composition series:
  $1 = G_0 \trianglelefteq G_1 \trianglelefteq \cdots \trianglelefteq G_{n-1} \trianglelefteq G_n = G$ where G_{i+1}/G_i is simple.
- Jordan-Hölder: Uniqueness of subquotients.
- Classification of finite simple groups.

Infinite

- Adian-Rabin '55:
  The isomorphism problem for finitely presented groups is undecidable.
- Olshansky, Vaughan-Lee '70:
  There exist continuously many different varieties of groups.

Let G be a group.

Finite

- Composition series:
  $1 = G_0 \trianglelefteq G_1 \trianglelefteq \cdots \trianglelefteq G_{n-1} \trianglelefteq G_n = G$ where G_{i+1}/G_i is simple.
- Jordan-Hölder: Uniqueness of subquotients.
- Classification of finite simple groups.

Infinite

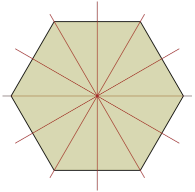
- Adian-Rabin '55:
  The isomorphism problem for finitely presented groups is undecidable.
- Olshansky, Vaughan-Lee '70:
  There exist continuously many different varieties of groups.
  (closed under homomorphic images, subgroups, cartesian products)

Let G be a locally compact group.

Let G be a locally compact group. For example...

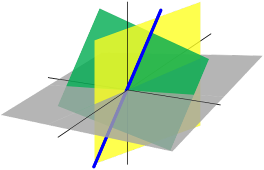
Let G be a locally compact group. For example...

Group Theory



D_6

Linear Algebra



$\mathrm{GL}(\mathrm{n}, \mathbb{R})$

Number Theory

$K \subseteq E$ fields

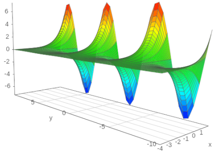
$\mathbb{Q} \subseteq \mathbb{Q}(\sqrt{2}, \sqrt{3})$

$\mathbb{F}_p \subseteq \overline{\mathbb{F}_p(X)}$

$\mathrm{Aut}(E/K)$

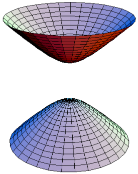
Differential Equations

$\Delta f = 0$



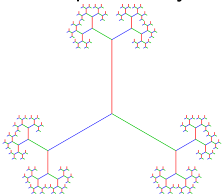
$\mathrm{O}(n)$

Differential Geometry



$\mathrm{O}(1, n)$

Graph Theory



$\mathrm{Aut}(T_d)$

Let G be a locally compact group.

Let G be a locally compact group.

$$G$$

Let G be a locally compact group.

$$1 \rightarrowtail G^0 \rightarrowtail G$$

Let G be a locally compact group.

$$1 \rightarrowtail G^0 \xrightarrow[\textit{normal}]{\textit{closed}} G$$

Let G be a locally compact group.

$$1 \rightarrowtail G^0 \underset{\textit{normal}}{\overset{\textit{closed}}{\rightarrowtail}} G \twoheadrightarrow G/G_0 \twoheadrightarrow 1$$

Let G be a locally compact group.

$$1 \rightarrowtail G^0 \xrightarrow[\textit{normal}]{\textit{closed}} G \twoheadrightarrow G/G_0 \twoheadrightarrow 1$$

$$D_6$$

Let G be a locally compact group.

$$1 \rightarrowtail G^0 \overset{closed}{\underset{normal}{\rightarrowtail}} G \twoheadrightarrow G/G_0 \twoheadrightarrow 1$$

$$1 \rightarrowtail 1 \rightarrowtail D_6$$

Let G be a locally compact group.

$$1 \rightarrowtail G^0 \xrightarrow[\text{normal}]{\text{closed}} G \twoheadrightarrow G/G_0 \twoheadrightarrow 1$$

$$1 \rightarrowtail 1 \rightarrowtail D_6 \twoheadrightarrow D_6 \twoheadrightarrow 1$$

Let G be a locally compact group.

$$1 \rightarrowtail G^0 \xrightarrow[\textit{normal}]{\textit{closed}} G \twoheadrightarrow G/G_0 \twoheadrightarrow 1$$

$$1 \rightarrowtail 1 \rightarrowtail D_6 \twoheadrightarrow D_6 \twoheadrightarrow 1$$

$$\mathrm{GL}(n, \mathbb{R})$$

Let G be a locally compact group.

$$1 \rightarrowtail G^0 \overset{\textit{closed}}{\underset{\textit{normal}}{\rightarrowtail}} G \twoheadrightarrow G/G_0 \twoheadrightarrow 1$$

$$1 \rightarrowtail 1 \rightarrowtail D_6 \twoheadrightarrow D_6 \twoheadrightarrow 1$$

$$1 \rightarrowtail \mathrm{GL}^+(n,\mathbb{R}) \rightarrowtail \mathrm{GL}(n,\mathbb{R})$$

Let G be a locally compact group.

$$1 \rightarrowtail G^0 \xrightarrow[\textit{normal}]{\textit{closed}} G \twoheadrightarrow G/G_0 \twoheadrightarrow 1$$

$$1 \rightarrowtail 1 \rightarrowtail D_6 \twoheadrightarrow D_6 \twoheadrightarrow 1$$

$$1 \rightarrowtail \mathrm{GL}^+(n, \mathbb{R}) \rightarrowtail \mathrm{GL}(n, \mathbb{R}) \twoheadrightarrow \{\pm 1\} \twoheadrightarrow 1$$

Let G be a locally compact group.

$$1 \rightarrowtail G^0 \xrightarrow[\textit{normal}]{\textit{closed}} G \twoheadrightarrow G/G_0 \twoheadrightarrow 1$$

$$1 \rightarrowtail 1 \rightarrowtail D_6 \twoheadrightarrow D_6 \twoheadrightarrow 1$$

$$1 \rightarrowtail \mathrm{GL}^+(n,\mathbb{R}) \rightarrowtail \mathrm{GL}(n,\mathbb{R}) \twoheadrightarrow \{\pm 1\} \twoheadrightarrow 1$$

$$\mathrm{Aut}(E/K)$$

Let G be a locally compact group.

$$1 \rightarrowtail G^0 \overset{\textit{closed}}{\underset{\textit{normal}}{\rightarrowtail}} G \twoheadrightarrow G/G_0 \twoheadrightarrow 1$$

$$1 \rightarrowtail 1 \rightarrowtail D_6 \twoheadrightarrow D_6 \twoheadrightarrow 1$$

$$1 \rightarrowtail \mathrm{GL}^+(n,\mathbb{R}) \rightarrowtail \mathrm{GL}(n,\mathbb{R}) \twoheadrightarrow \{\pm 1\} \twoheadrightarrow 1$$

$$1 \rightarrowtail 1 \rightarrowtail \mathrm{Aut}(E/K) \twoheadrightarrow \mathrm{Aut}(E/K) \twoheadrightarrow 1$$

Let G be a locally compact group.

$$1 \rightarrowtail G^0 \xrightarrow[\textit{normal}]{\textit{closed}} G \twoheadrightarrow G/G_0 \twoheadrightarrow 1$$

$$1 \rightarrowtail 1 \rightarrowtail D_6 \twoheadrightarrow D_6 \twoheadrightarrow 1$$

$$1 \rightarrowtail \mathrm{GL}^+(n,\mathbb{R}) \rightarrowtail \mathrm{GL}(n,\mathbb{R}) \twoheadrightarrow \{\pm 1\} \twoheadrightarrow 1$$

$$1 \rightarrowtail 1 \rightarrowtail \mathrm{Aut}(E/K) \twoheadrightarrow \mathrm{Aut}(E/K) \twoheadrightarrow 1$$

$$\mathrm{O}(n)$$

Let G be a locally compact group.

$$1 \rightarrowtail G^0 \overset{\textit{closed}}{\underset{\textit{normal}}{\rightarrowtail}} G \twoheadrightarrow G/G_0 \twoheadrightarrow 1$$

$$1 \rightarrowtail 1 \rightarrowtail D_6 \twoheadrightarrow D_6 \twoheadrightarrow 1$$

$$1 \rightarrowtail \mathrm{GL}^+(n,\mathbb{R}) \rightarrowtail \mathrm{GL}(n,\mathbb{R}) \twoheadrightarrow \{\pm 1\} \twoheadrightarrow 1$$

$$1 \rightarrowtail 1 \rightarrowtail \mathrm{Aut}(E/K) \twoheadrightarrow \mathrm{Aut}(E/K) \twoheadrightarrow 1$$

$$1 \rightarrowtail \mathrm{SO}(n) \rightarrowtail \mathrm{O}(n)$$

Let G be a locally compact group.

$$1 \rightarrowtail G^0 \underset{\textit{normal}}{\overset{\textit{closed}}{\rightarrowtail}} G \twoheadrightarrow G/G_0 \twoheadrightarrow 1$$

$$1 \rightarrowtail 1 \rightarrowtail D_6 \twoheadrightarrow D_6 \twoheadrightarrow 1$$

$$1 \rightarrowtail \mathrm{GL}^+(n,\mathbb{R}) \rightarrowtail \mathrm{GL}(n,\mathbb{R}) \twoheadrightarrow \{\pm 1\} \twoheadrightarrow 1$$

$$1 \rightarrowtail 1 \rightarrowtail \mathrm{Aut}(E/K) \twoheadrightarrow \mathrm{Aut}(E/K) \twoheadrightarrow 1$$

$$1 \rightarrowtail \mathrm{SO}(n) \rightarrowtail \mathrm{O}(n) \twoheadrightarrow \{\pm 1\} \twoheadrightarrow 1$$

Let G be a locally compact group.

$$1 \rightarrowtail G^0 \xrightarrow[\textit{normal}]{\textit{closed}} G \twoheadrightarrow G/G_0 \twoheadrightarrow 1$$

$$1 \rightarrowtail 1 \rightarrowtail D_6 \twoheadrightarrow D_6 \twoheadrightarrow 1$$

$$1 \rightarrowtail \mathrm{GL}^+(n,\mathbb{R}) \rightarrowtail \mathrm{GL}(n,\mathbb{R}) \twoheadrightarrow \{\pm 1\} \twoheadrightarrow 1$$

$$1 \rightarrowtail 1 \rightarrowtail \mathrm{Aut}(E/K) \twoheadrightarrow \mathrm{Aut}(E/K) \twoheadrightarrow 1$$

$$1 \rightarrowtail \mathrm{SO}(n) \rightarrowtail \mathrm{O}(n) \twoheadrightarrow \{\pm 1\} \twoheadrightarrow 1$$

$$\mathrm{O}(1,n)$$

Let G be a locally compact group.

$$1 \rightarrowtail G^0 \underset{normal}{\overset{closed}{\rightarrowtail}} G \twoheadrightarrow G/G_0 \twoheadrightarrow 1$$

$$1 \rightarrowtail 1 \rightarrowtail D_6 \twoheadrightarrow D_6 \twoheadrightarrow 1$$

$$1 \rightarrowtail \mathrm{GL}^+(n, \mathbb{R}) \rightarrowtail \mathrm{GL}(n, \mathbb{R}) \twoheadrightarrow \{\pm 1\} \twoheadrightarrow 1$$

$$1 \rightarrowtail 1 \rightarrowtail \mathrm{Aut}(E/K) \twoheadrightarrow \mathrm{Aut}(E/K) \twoheadrightarrow 1$$

$$1 \rightarrowtail \mathrm{SO}(n) \rightarrowtail \mathrm{O}(n) \twoheadrightarrow \{\pm 1\} \twoheadrightarrow 1$$

$$1 \rightarrowtail \mathrm{SO}_0(1, n) \rightarrowtail \mathrm{O}(1, n)$$

Let G be a locally compact group.

$$1 \rightarrowtail G^0 \xrightarrow[\textit{normal}]{\textit{closed}} G \twoheadrightarrow G/G_0 \twoheadrightarrow 1$$

$$1 \rightarrowtail 1 \rightarrowtail D_6 \twoheadrightarrow D_6 \twoheadrightarrow 1$$

$$1 \rightarrowtail \mathrm{GL}^+(n,\mathbb{R}) \rightarrowtail \mathrm{GL}(n,\mathbb{R}) \twoheadrightarrow \{\pm 1\} \twoheadrightarrow 1$$

$$1 \rightarrowtail 1 \rightarrowtail \mathrm{Aut}(E/K) \twoheadrightarrow \mathrm{Aut}(E/K) \twoheadrightarrow 1$$

$$1 \rightarrowtail \mathrm{SO}(n) \rightarrowtail \mathrm{O}(n) \twoheadrightarrow \{\pm 1\} \twoheadrightarrow 1$$

$$1 \rightarrowtail \mathrm{SO}_0(1,n) \rightarrowtail \mathrm{O}(1,n) \twoheadrightarrow \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z} \twoheadrightarrow 1$$

Let G be a locally compact group.

$$1 \rightarrowtail G^0 \underset{\text{normal}}{\overset{\text{closed}}{\rightarrowtail}} G \twoheadrightarrow G/G_0 \twoheadrightarrow 1$$

$$1 \rightarrowtail 1 \rightarrowtail D_6 \twoheadrightarrow D_6 \twoheadrightarrow 1$$

$$1 \rightarrowtail \mathrm{GL}^+(n,\mathbb{R}) \rightarrowtail \mathrm{GL}(n,\mathbb{R}) \twoheadrightarrow \{\pm 1\} \twoheadrightarrow 1$$

$$1 \rightarrowtail 1 \rightarrowtail \mathrm{Aut}(E/K) \twoheadrightarrow \mathrm{Aut}(E/K) \twoheadrightarrow 1$$

$$1 \rightarrowtail \mathrm{SO}(n) \rightarrowtail \mathrm{O}(n) \twoheadrightarrow \{\pm 1\} \twoheadrightarrow 1$$

$$1 \rightarrowtail \mathrm{SO}_0(1,n) \rightarrowtail \mathrm{O}(1,n) \twoheadrightarrow \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z} \twoheadrightarrow 1$$

$$\mathrm{Aut}(T_d)$$

Let G be a locally compact group.

$$1 \rightarrowtail G^0 \underset{normal}{\overset{closed}{\rightarrowtail}} G \twoheadrightarrow G/G_0 \twoheadrightarrow 1$$

$$1 \rightarrowtail 1 \rightarrowtail D_6 \twoheadrightarrow D_6 \twoheadrightarrow 1$$

$$1 \rightarrowtail \mathrm{GL}^+(n,\mathbb{R}) \rightarrowtail \mathrm{GL}(n,\mathbb{R}) \twoheadrightarrow \{\pm 1\} \twoheadrightarrow 1$$

$$1 \rightarrowtail 1 \rightarrowtail \mathrm{Aut}(E/K) \twoheadrightarrow \mathrm{Aut}(E/K) \twoheadrightarrow 1$$

$$1 \rightarrowtail \mathrm{SO}(n) \rightarrowtail \mathrm{O}(n) \twoheadrightarrow \{\pm 1\} \twoheadrightarrow 1$$

$$1 \rightarrowtail \mathrm{SO}_0(1,n) \rightarrowtail \mathrm{O}(1,n) \twoheadrightarrow \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z} \twoheadrightarrow 1$$

$$1 \rightarrowtail 1 \rightarrowtail \mathrm{Aut}(T_d) \twoheadrightarrow \mathrm{Aut}(T_d) \twoheadrightarrow 1$$

Let G be a locally compact group.

$$1 \rightarrowtail G^0 \rightarrowtail G$$

Let G be a locally compact group.
Every connected locally compact group is an inverse limit of Lie groups.

$$1 \rightarrowtail G^0 \rightarrowtail G$$

Let G be a locally compact group.
Every connected locally compact group is an inverse limit of Lie groups.
(Hilbert's 5th problem; Gleason, Yamabe, Montgomery-Zippin; 50's)

$$1 \rightarrowtail G^0 \rightarrowtail G$$

Let G be a locally compact group.
Every connected locally compact group is an inverse limit of Lie groups.
(Hilbert's 5th problem; Gleason, Yamabe, Montgomery-Zippin; 50's)

$$1 \rightarrowtail G^0 \xrightarrow[\text{normal}]{\text{closed}} G$$

Let G be a locally compact group.
Every connected locally compact group is an inverse limit of Lie groups.
(Hilbert's 5th problem; Gleason, Yamabe, Montgomery-Zippin; 50's)

$$1 \rightarrowtail G^0 \underset{\text{normal}}{\overset{\text{closed}}{\rightarrowtail}} G \twoheadrightarrow G/G^0 \twoheadrightarrow 1$$

Let G be a locally compact group.
Every connected locally compact group is an inverse limit of Lie groups.
(Hilbert's 5th problem; Gleason, Yamabe, Montgomery-Zippin; 50's)

Theorem (Abels '73, inspired by Cayley, Schreier)

Let G be a totally disconnected locally compact group.

$$1 \rightarrowtail G^0 \xrightarrow[\text{normal}]{\text{closed}} G \twoheadrightarrow G/G^0 \twoheadrightarrow 1$$

Let G be a locally compact group.
Every connected locally compact group is an inverse limit of Lie groups.
(Hilbert's 5th problem; Gleason, Yamabe, Montgomery-Zippin; 50's)

Theorem (Abels '73, inspired by Cayley, Schreier)

Let G be a totally disconnected locally compact group.
Then G acts vertex-transitively on a connected, locally finite graph Γ with compact open vertex stabilisers **if and only if** *G is compactly generated.*

$$1 \rightarrowtail G^0 \xrightarrow[\text{normal}]{\text{closed}} G \twoheadrightarrow G/G^0 \twoheadrightarrow 1$$

Let G be a locally compact group.
Every connected locally compact group is an inverse limit of Lie groups.
(Hilbert's 5th problem; Gleason, Yamabe, Montgomery-Zippin; 50's)

Theorem (Abels '73, inspired by Cayley, Schreier)

Let G be a totally disconnected locally compact group.
Then G acts vertex-transitively on a connected, locally finite graph Γ with compact open vertex stabilisers **if and only if** *G is compactly generated.*

D_6

$$1 \rightarrowtail G^0 \xrightarrow[\text{normal}]{\text{closed}} G \twoheadrightarrow G/G^0 \twoheadrightarrow 1$$

Let G be a locally compact group.
Every connected locally compact group is an inverse limit of Lie groups.
(Hilbert's 5th problem; Gleason, Yamabe, Montgomery-Zippin; 50's)

Theorem (Abels '73, inspired by Cayley, Schreier)

Let G be a totally disconnected locally compact group.
Then G acts vertex-transitively on a connected, locally finite graph Γ with compact open vertex stabilisers **if and only if** *G is compactly generated.*

$D_6 \curvearrowright$ Hexagon;

$$1 \rightarrowtail G^0 \underset{\text{normal}}{\overset{\text{closed}}{\rightarrowtail}} G \twoheadrightarrow G/G^0 \twoheadrightarrow 1$$

Let G be a locally compact group.
Every connected locally compact group is an inverse limit of Lie groups.
(Hilbert's 5th problem; Gleason, Yamabe, Montgomery-Zippin; 50's)

Theorem (Abels '73, inspired by Cayley, Schreier)

Let G be a totally disconnected locally compact group.
Then G acts vertex-transitively on a connected, locally finite graph Γ with compact open vertex stabilisers **if and only if** *G is compactly generated.*

$D_6 \curvearrowright$ Hexagon; $\mathrm{Aut}(\overline{\mathbb{Q}}/\mathbb{Q})$

$$1 \rightarrowtail G^0 \xrightarrow[\text{normal}]{\text{closed}} G \twoheadrightarrow G/G^0 \twoheadrightarrow 1$$

Let G be a locally compact group.
Every connected locally compact group is an inverse limit of Lie groups.
(Hilbert's 5th problem; Gleason, Yamabe, Montgomery-Zippin; 50's)

Theorem (Abels '73, inspired by Cayley, Schreier)

Let G be a totally disconnected locally compact group.
Then G acts vertex-transitively on a connected, locally finite graph Γ with compact open vertex stabilisers **if and only if** *G is compactly generated.*

$D_6 \curvearrowright$ Hexagon; $\mathrm{Aut}(\overline{\mathbb{Q}}/\mathbb{Q}) \curvearrowright \bullet$;

$$1 \rightarrowtail G^0 \xrightarrow[\text{normal}]{\text{closed}} G \twoheadrightarrow G/G^0 \twoheadrightarrow 1$$

Let G be a locally compact group.
Every connected locally compact group is an inverse limit of Lie groups.
(Hilbert's 5th problem; Gleason, Yamabe, Montgomery-Zippin; 50's)

Theorem (Abels '73, inspired by Cayley, Schreier)

Let G be a totally disconnected locally compact group.
Then G acts vertex-transitively on a connected, locally finite graph Γ with compact open vertex stabilisers **if and only if** *G is compactly generated.*

$D_6 \curvearrowright$ Hexagon; $\mathrm{Aut}(\overline{\mathbb{Q}}/\mathbb{Q}) \curvearrowright \bullet$; $\mathrm{Aut}(T_3)$

$$1 \rightarrowtail G^0 \overset{\text{closed}}{\underset{\text{normal}}{\rightarrowtail}} G \twoheadrightarrow G/G^0 \twoheadrightarrow 1$$

Let G be a locally compact group.
Every connected locally compact group is an inverse limit of Lie groups.
(Hilbert's 5th problem; Gleason, Yamabe, Montgomery-Zippin; 50's)

Theorem (Abels '73, inspired by Cayley, Schreier)

Let G be a totally disconnected locally compact group.
Then G acts vertex-transitively on a connected, locally finite graph Γ with compact open vertex stabilisers **if and only if** *G is compactly generated.*

$D_6 \curvearrowright$ Hexagon; $\mathrm{Aut}(\overline{\mathbb{Q}}/\mathbb{Q}) \curvearrowright \bullet$; $\mathrm{Aut}(T_3) \curvearrowright T_3$.

$$1 \rightarrowtail G^0 \underset{\text{normal}}{\overset{\text{closed}}{\rightarrowtail}} G \twoheadrightarrow G/G^0 \twoheadrightarrow 1$$

Let G be a locally compact group such that G/G^0 is compactly generated.
Every connected locally compact group is an inverse limit of Lie groups.
(Hilbert's 5th problem; Gleason, Yamabe, Montgomery-Zippin; 50's)

Theorem (Abels '73, inspired by Cayley, Schreier)

Let G be a totally disconnected locally compact group.
Then G acts vertex-transitively on a connected, locally finite graph Γ with compact open vertex stabilisers **if and only if** *G is compactly generated.*

$D_6 \curvearrowright$ Hexagon; $\mathrm{Aut}(\overline{\mathbb{Q}}/\mathbb{Q}) \curvearrowright \bullet$; $\mathrm{Aut}(T_3) \curvearrowright T_3$.

$$1 \rightarrowtail G^0 \overset{\text{closed}}{\underset{\text{normal}}{\rightarrowtail}} G \twoheadrightarrow G/G^0 \twoheadrightarrow 1$$

Let G be a locally compact group such that G/G^0 is compactly generated. Every connected locally compact group is an inverse limit of Lie groups. (Hilbert's 5th problem; Gleason, Yamabe, Montgomery-Zippin; 50's)

Theorem (Abels '73, inspired by Cayley, Schreier)

Let G be a totally disconnected locally compact group. Then G acts vertex-transitively on a connected, locally finite graph Γ with compact open vertex stabilisers **if and only if** *G is compactly generated.*

$D_6 \curvearrowright$ Hexagon; $\mathrm{Aut}(\overline{\mathbb{Q}}/\mathbb{Q}) \curvearrowright \bullet$; $\mathrm{Aut}(T_3) \curvearrowright T_3$.

$$1 \rightarrowtail G^0 \underset{\text{normal}}{\overset{\text{closed}}{\rightarrowtail}} G \twoheadrightarrow G/G^0 \twoheadrightarrow 1$$

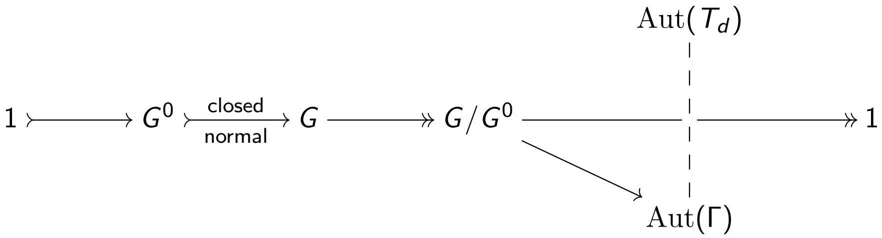
$$G/G^0 \to \mathrm{Aut}(\Gamma)$$

Let G be a locally compact group such that G/G^0 is compactly generated.

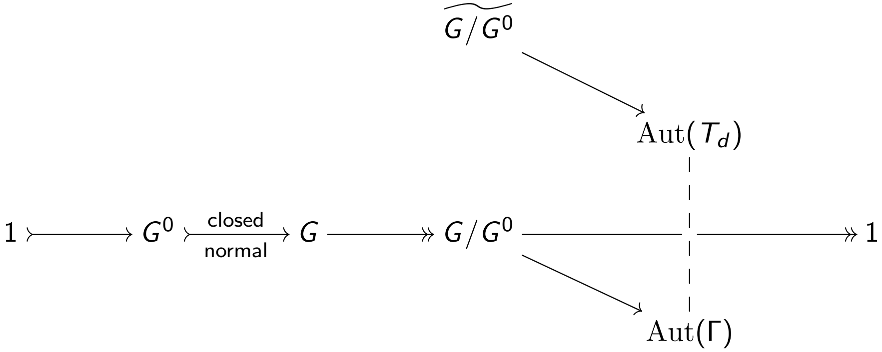
$$1 \rightarrowtail G^0 \xrightarrow[\text{normal}]{\text{closed}} G \twoheadrightarrow G/G^0 \twoheadrightarrow 1$$

$$G/G^0 \longrightarrow \mathrm{Aut}(\Gamma)$$

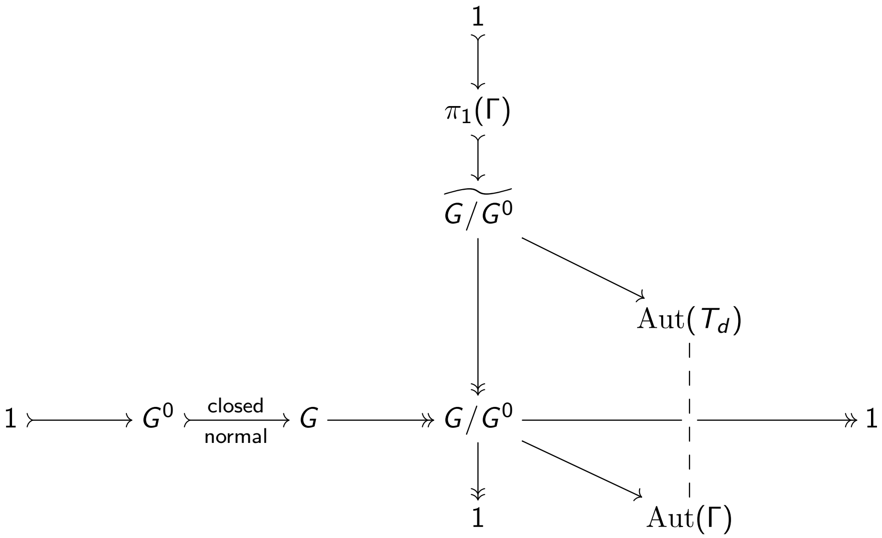
Let G be a locally compact group such that G/G^0 is compactly generated.



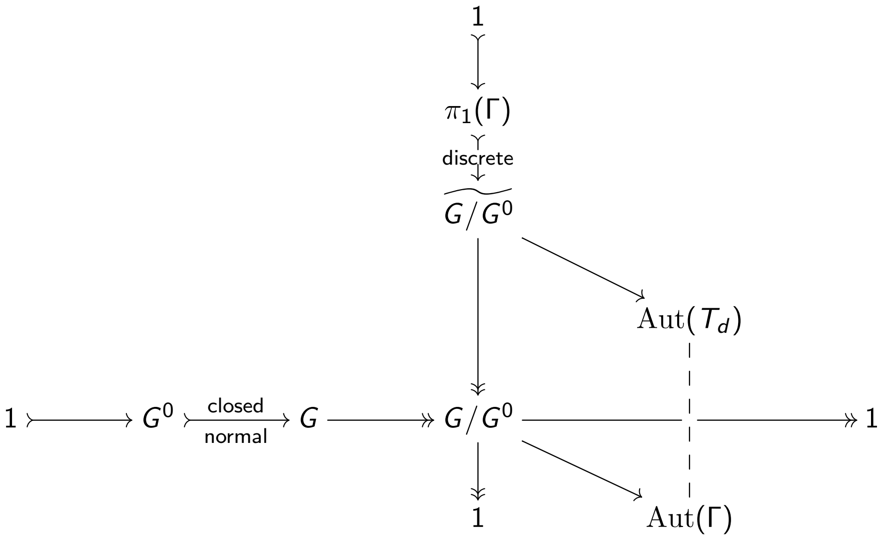
Let G be a locally compact group such that G/G^0 is compactly generated.



Let G be a locally compact group such that G/G^0 is compactly generated.



Let G be a locally compact group such that G/G^0 is compactly generated.



Let G be a locally compact group such that G/G^0 is compactly generated.