Groups acting on trees: constructions, computations and classifications

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Summary

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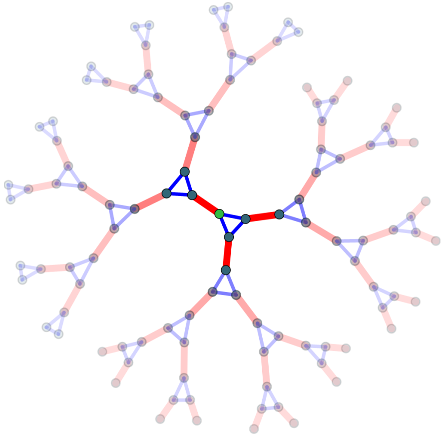

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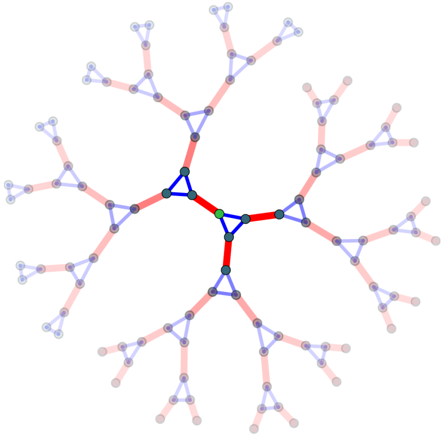

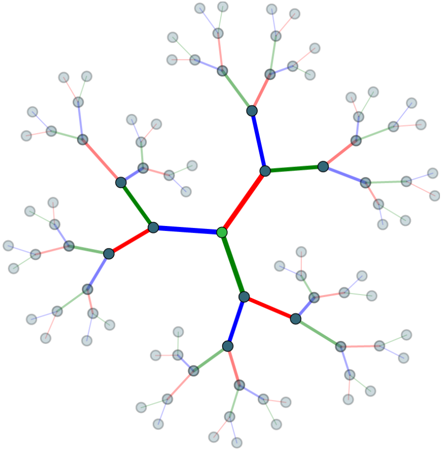

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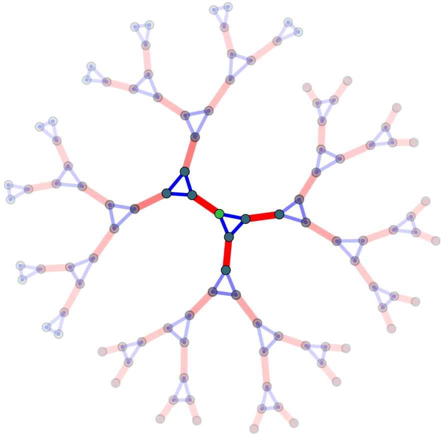

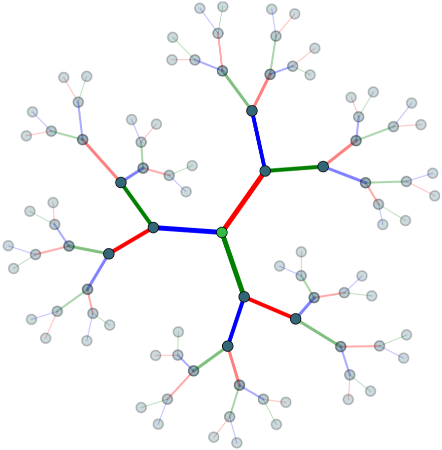

https://zerodimensional.group

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  (closed under homomorphic images, subgroups, cartesian products)

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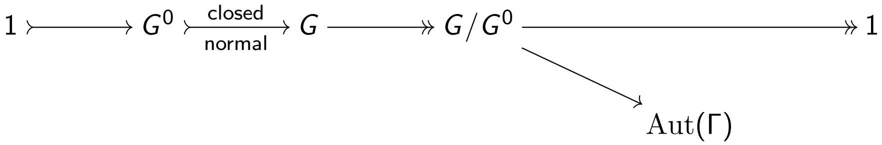

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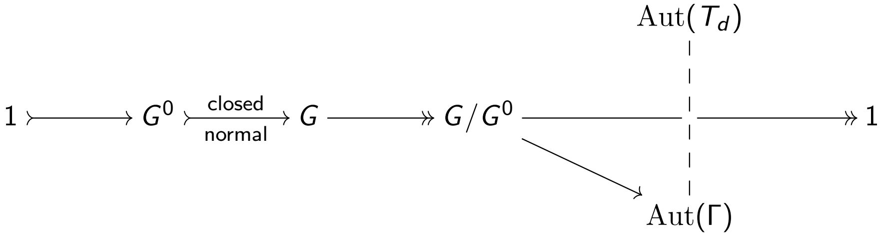

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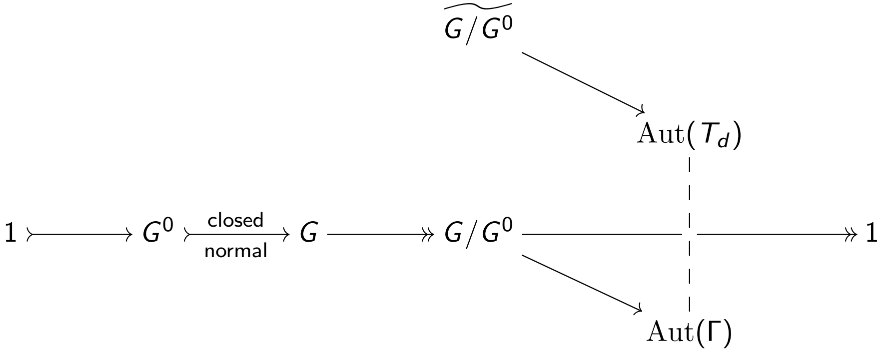

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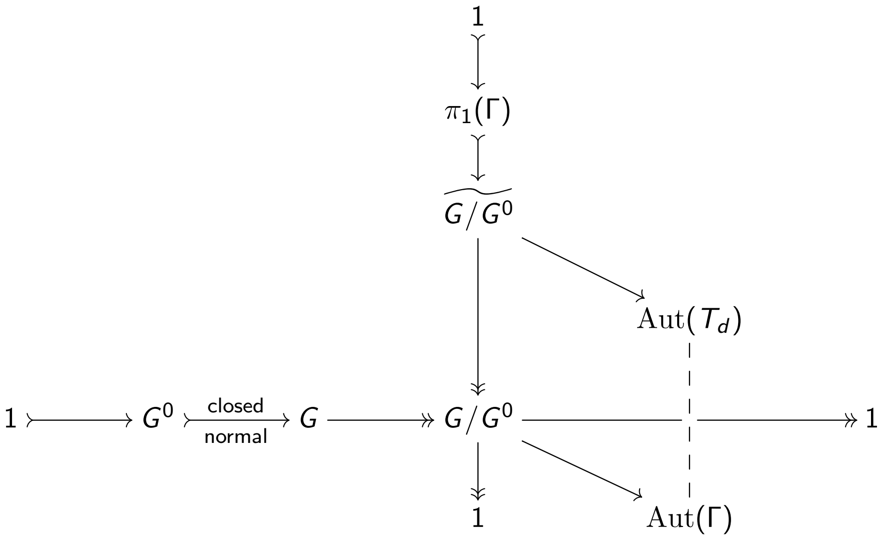

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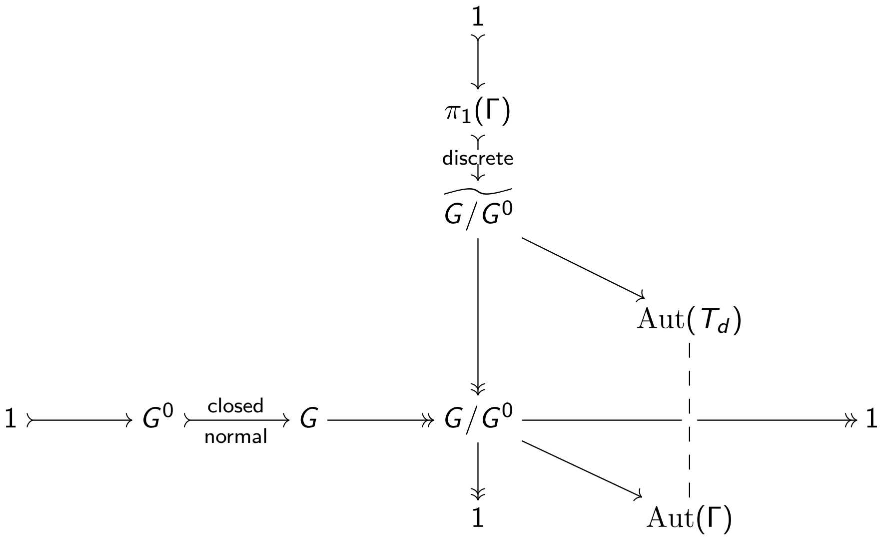

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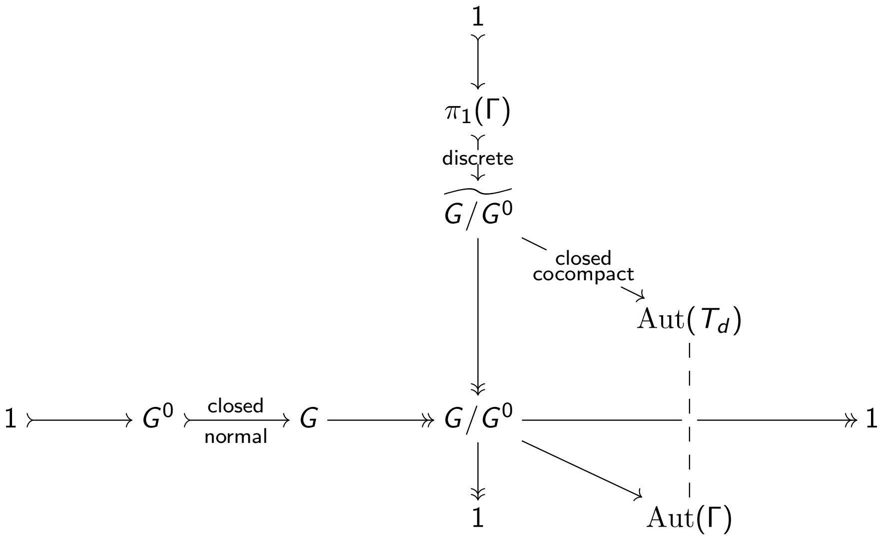

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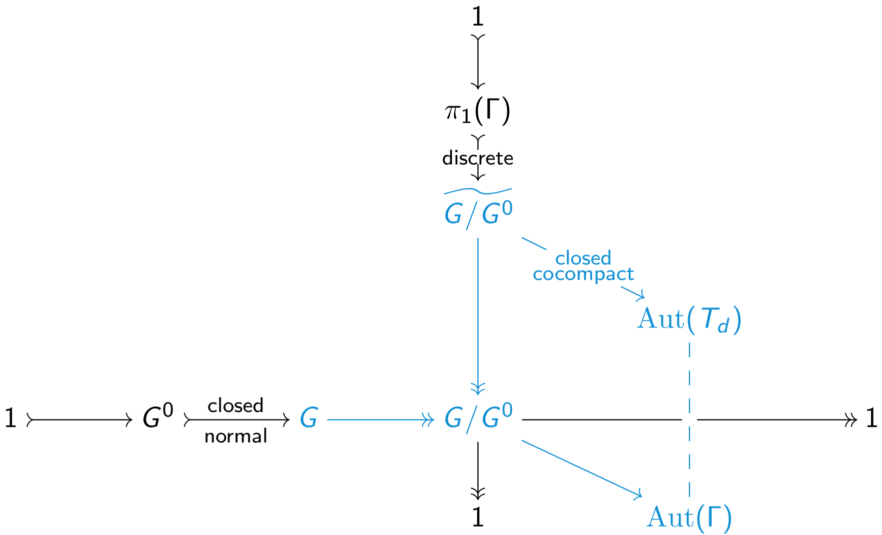

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3. *every* closed normal subgroup $N \trianglelefteq H$ *is either non-discrete cocompact and $N \trianglerighteq H^{(\infty)}$, or discrete and $N \trianglelefteq \mathrm{QZ}(H)$.*

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4. $H^{(\infty)}/\mathrm{QZ}(H^{(\infty)})$ *admits non-trivial, minimal closed normal subgroups; finitely many, H-conjugate and topologically simple.*

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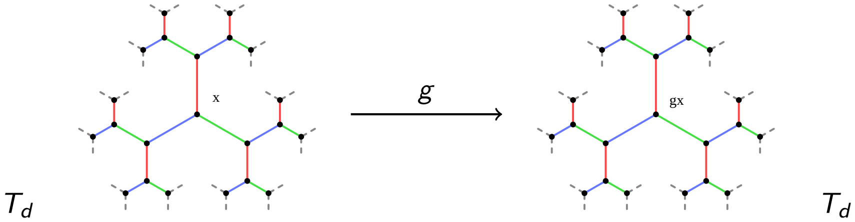

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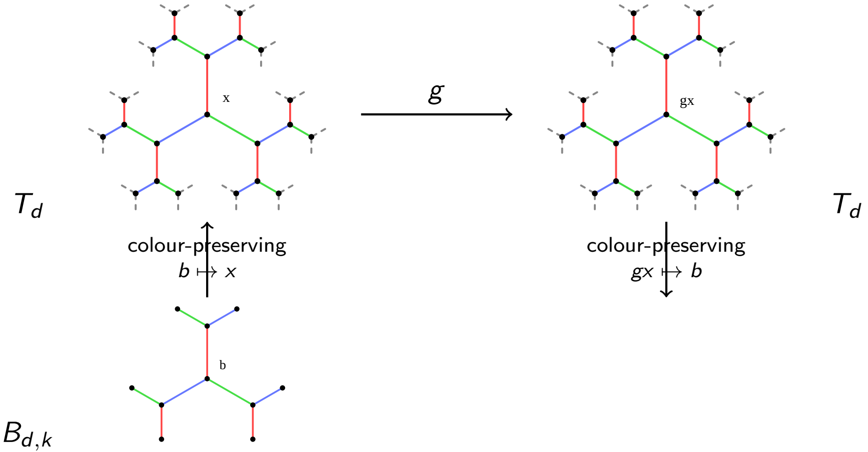

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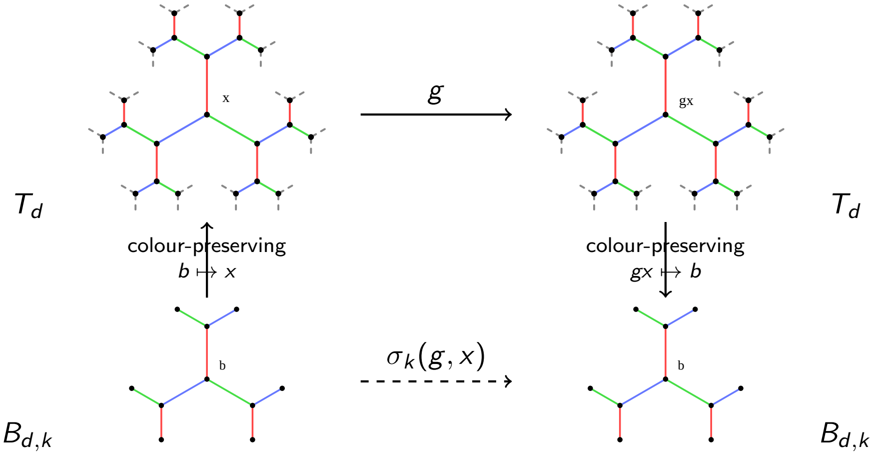

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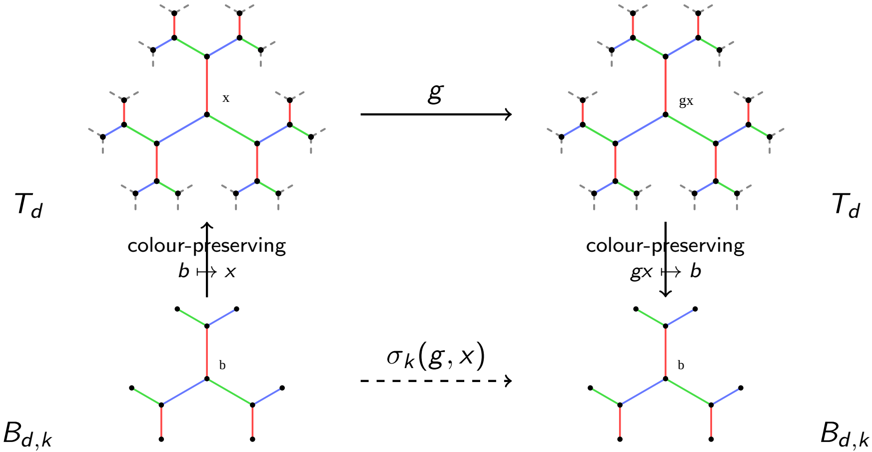

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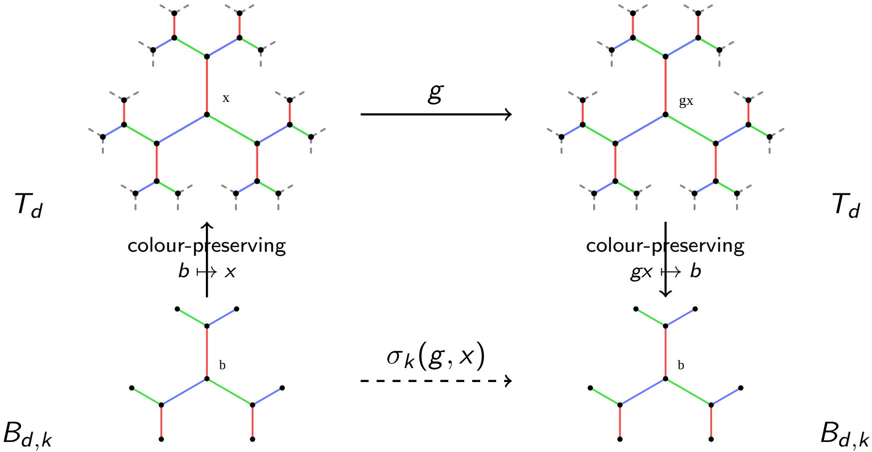

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For $F \leq \mathrm{Aut}(B_{d,k})$, set $\mathrm{U}_k(F) := \{g \in \mathrm{Aut}(T_d) \mid \forall x \in V(T_d) : \sigma_k(g, x) \in F\}$.

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1. closed in $\mathrm{Aut}(T_d)$,
2. vertex-transitive, and
3. compactly generated.

Question

Let $F \leq \mathrm{Aut}(B_{d,k})$. For $x \in V(T_d)$, what is the action that $\mathrm{U}_k(F)_x$ induces on $B(x,k)$?

The compatibility condition (C)

Definition

Let $F \leq \mathrm{Aut}(B_{d,k})$. Then $\mathrm{U}_k(F) \leq \mathrm{Aut}(T_d)$ satisfies (C) if and only if for all $x \in V(T_d)$ the actions $\mathrm{U}_k(F)_x \curvearrowright B(x,k)$ and $F \curvearrowright B_{d,k}$ are isomorphic.

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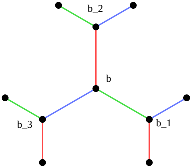

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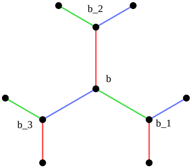

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$$g \mapsto (\sigma_{k-1}(g,b), (\sigma_{k-1}(g,b_1), \ldots, \sigma_{k-1}(g,b_d)))$$

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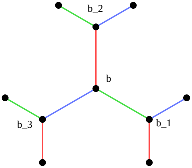

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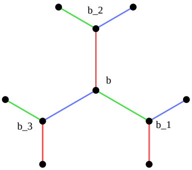

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$$\forall i \in \{1,\ldots,d\} \; \forall \, (\alpha,(\alpha_1,\ldots,\alpha_{i-1},\alpha_i,\alpha_{i+1},\ldots,\alpha_d)) \in F$$
$$\exists \, (\alpha_i,(\,?,\ldots,\;\;?\;\;,\alpha,\;\;?\;\;,\ldots,\,?\,)) \in F.$$

UGALY: A GAP package

Joint work with Khalil Hannouch.

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Create, analyse and find local actions of universal groups.

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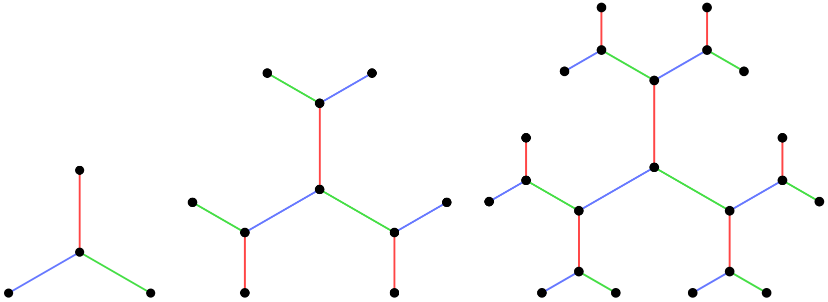

$B_{3,1}$ $B_{3,2}$ $B_{3,3}$

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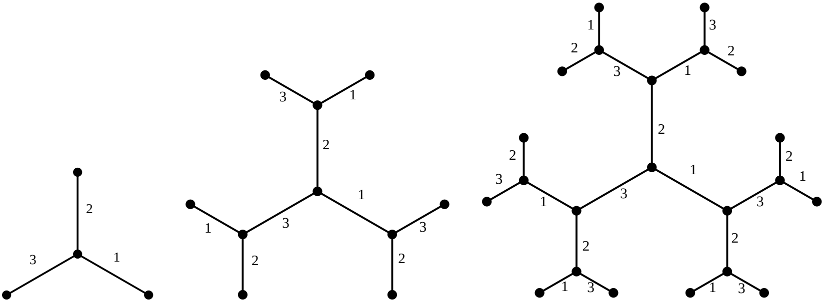

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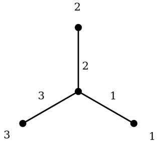
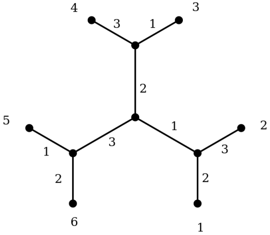
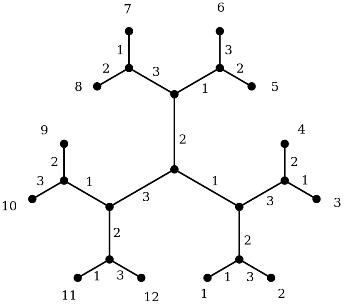


$B_{3,1}$



$B_{3,2}$



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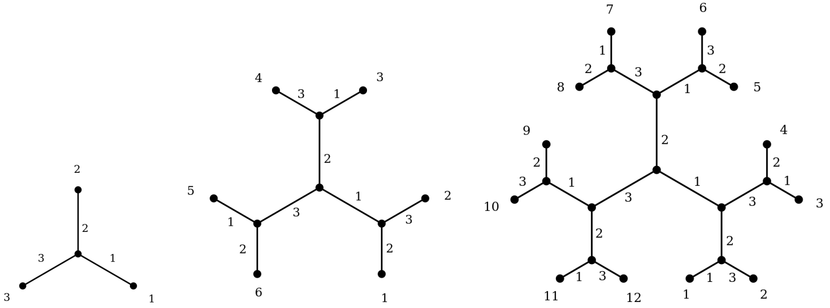

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github.com/torniers/UGALY

Towards a classification of closed vertex-transitive groups

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Proposition

Let $F \leq \mathrm{Aut}(B_{d,k})$. Then $\mathrm{U}_k(F) \leq \mathrm{Aut}(T_d)$ is (P_k)-closed.

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