Groups acting on trees from finite combinatorial data

Stephan Tornier

THE UNIVERSITY OF NEWCASTLE AUSTRALIA

June 11, 2021

Summary

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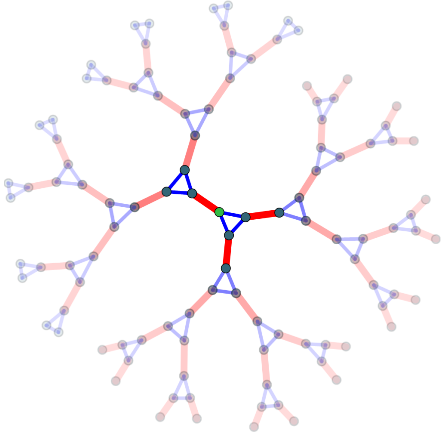

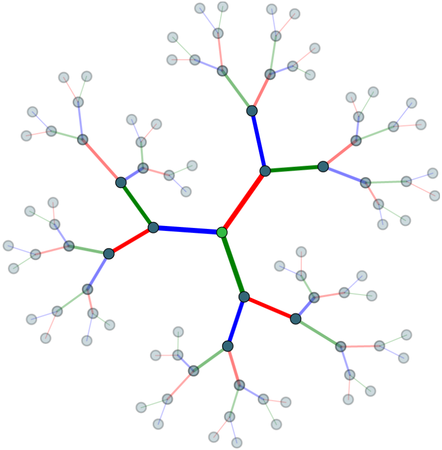

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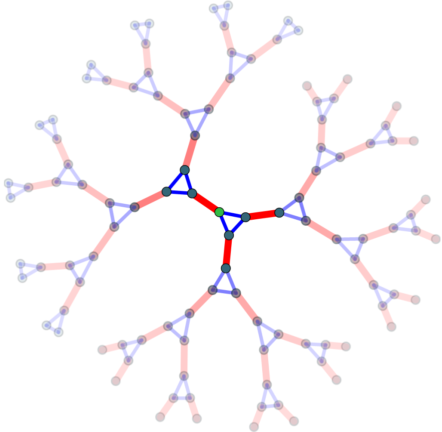

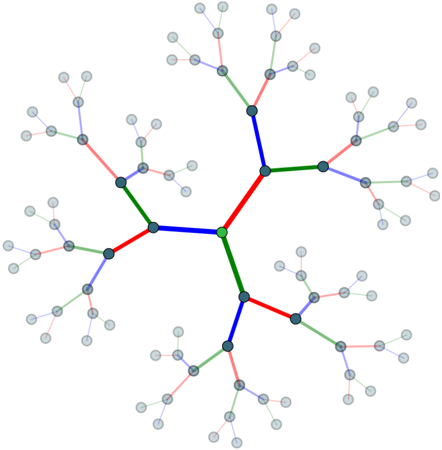

https://zerodimensional.group

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4. *$H^{(\infty)}/\mathrm{QZ}(H^{(\infty)})$ admits non-trivial, minimal closed normal subgroups; finitely many, H-conjugate and topologically simple.*

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Note: this theorem is essentially sharp.

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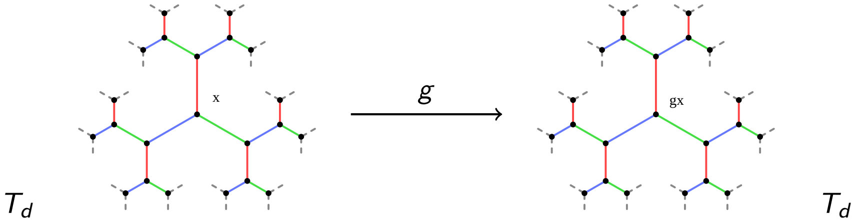

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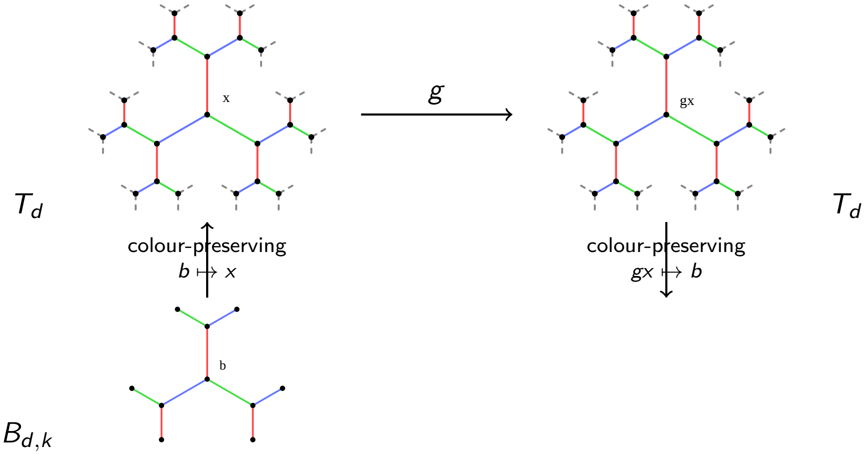

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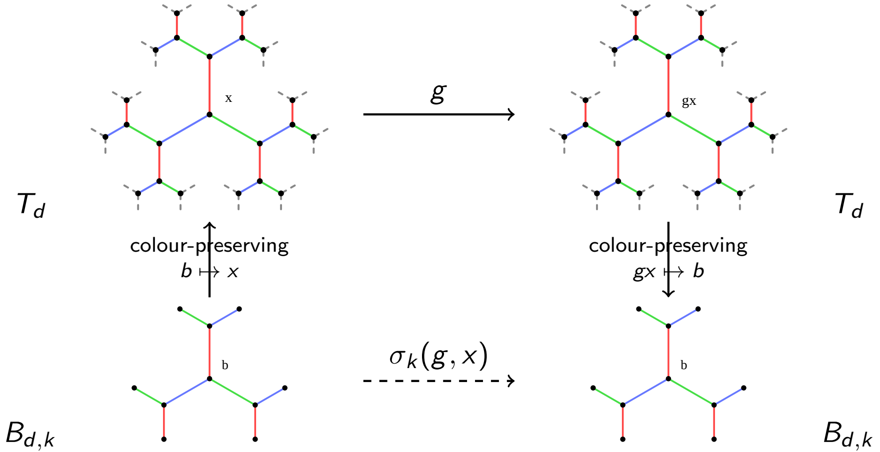

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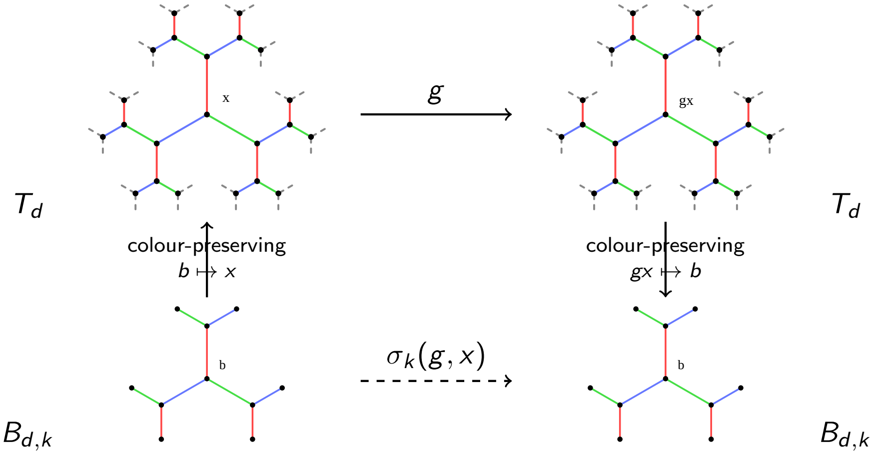

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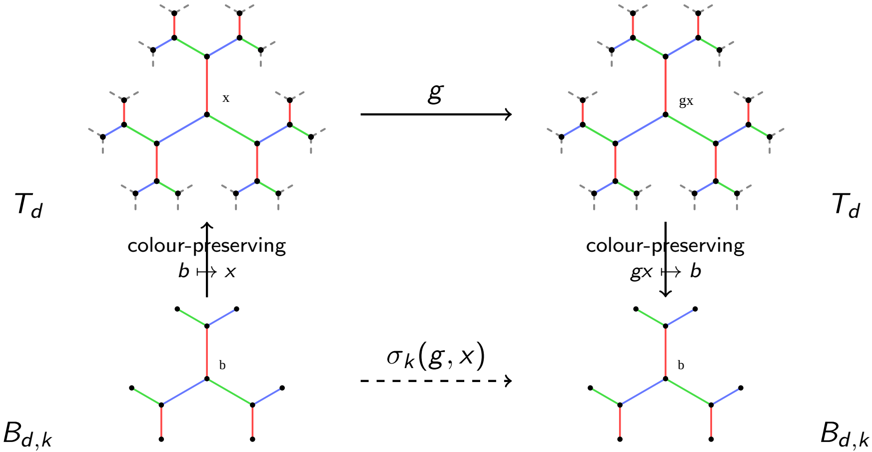

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Or: Let $F \leq \mathrm{Aut}(B_{d,k})$. What action does $\mathrm{U}_k(F)_x$ induce on $B(x,k)$?

The compatibility condition (C)

Definition

Let $F \leq \mathrm{Aut}(B_{d,k})$. Then F satisfies (C) if and only if for all $x \in V(T_d)$ the actions $\mathrm{U}_k(F)_x \curvearrowright B(x,k)$ and $F \curvearrowright B_{d,k}$ are isomorphic.

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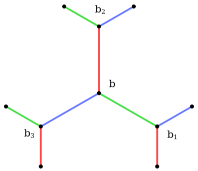

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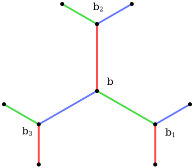

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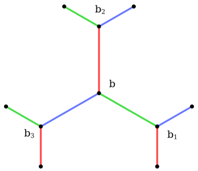

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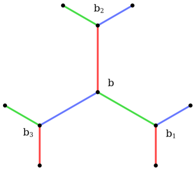

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$$\forall i \in \{1,\ldots,d\}\ \forall\, (\alpha, (\alpha_1, \ldots, \alpha_{i-1}, \alpha_i, \alpha_{i+1}, \ldots, \alpha_d)) \in F$$
$$\exists\, (\alpha_i, (\ ?, \ldots,\ \ ?\ \ , \alpha,\ \ ?\ \ , \ldots, ?\ )) \in F.$$

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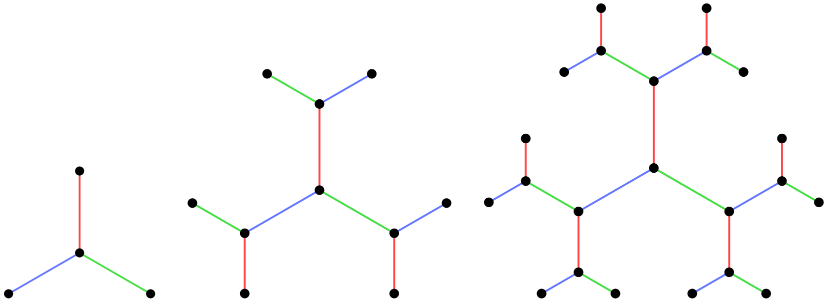

$B_{3,1}$ $B_{3,2}$ $B_{3,3}$

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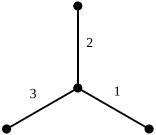

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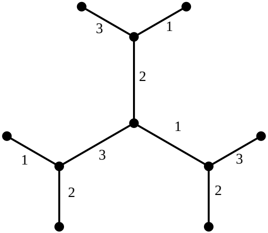

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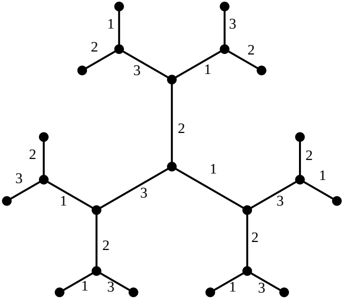

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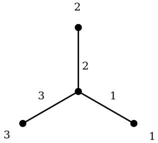

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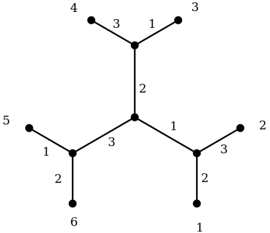

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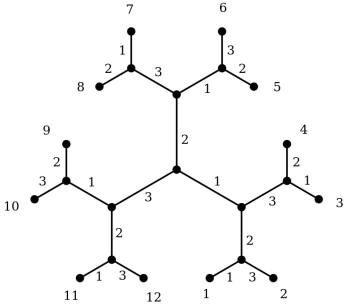

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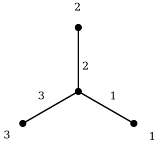

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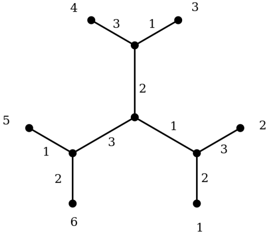

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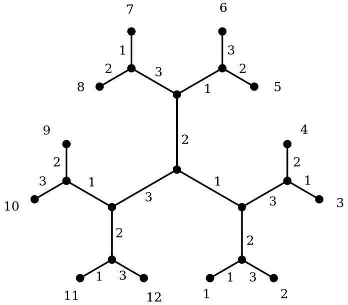

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github.com/torniers/UGALY

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$$H^{(\mathrm{P}_k)} := \{g \in \mathrm{Aut}(T_d) \mid \forall x \in V(T_d)\ \exists h \in H : g|_{B(x,k)} = h|_{B(x,k)}\}.$$

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$$\left\{ H \leq \mathrm{Aut}(T_d) \;\middle|\; \begin{array}{c} \textit{locally transitive} \\ \textit{inversion of order } 2 \\ \textit{Property } (P_k) \end{array} \right\} \overset{1:1}{\longleftrightarrow} \left\{ F \leq \mathrm{Aut}(B_{d,k}) \;\middle|\; \begin{array}{c} \textit{locally transitive} \\ \textit{Condition } (C) \end{array} \right\}$$

Construction II: (P_1)-closed groups

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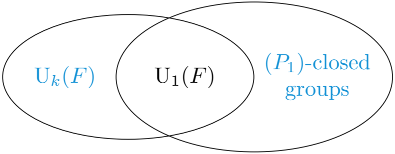

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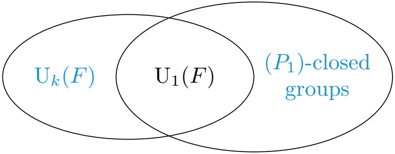

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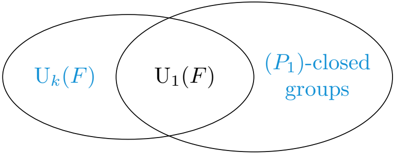

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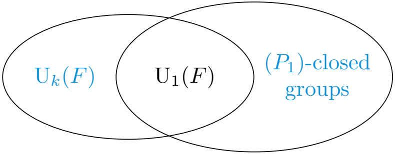

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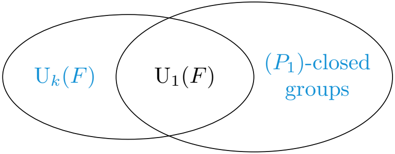

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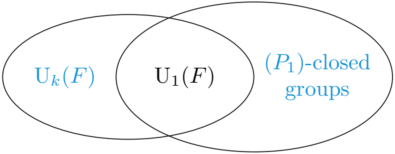

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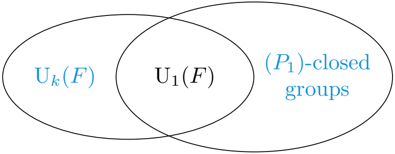

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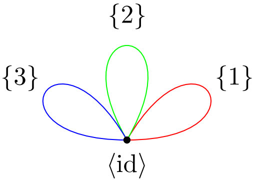

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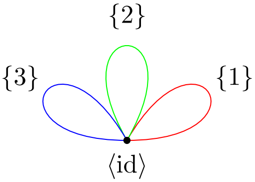

$\mathrm{U}_1(\{\mathrm{id}\})$

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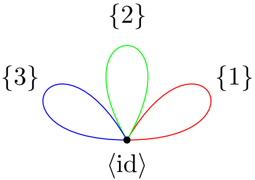

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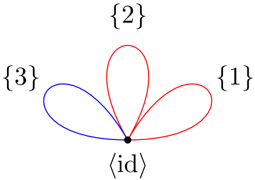

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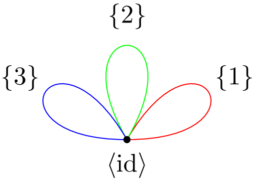

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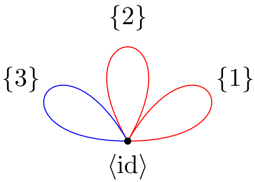

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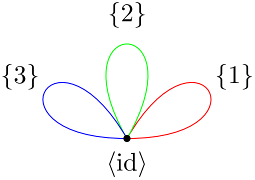

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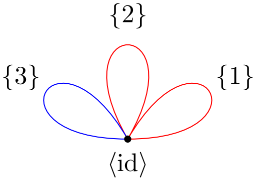

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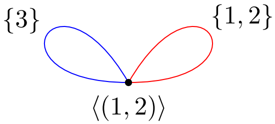

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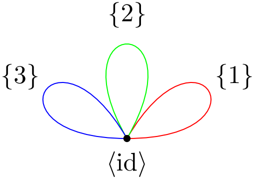

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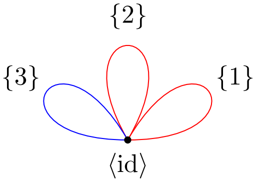

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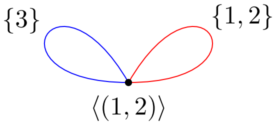

$\mathrm{U}_1(S_2)$

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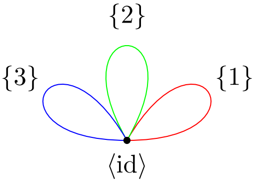

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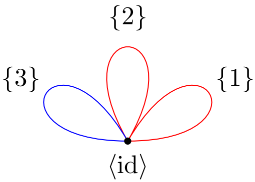

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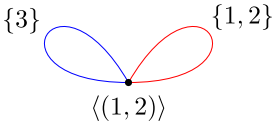

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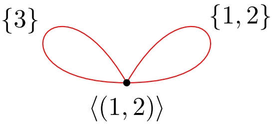

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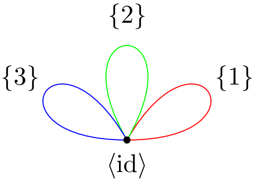

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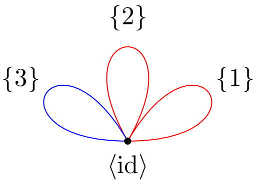

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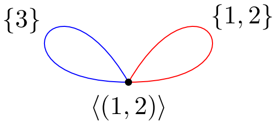

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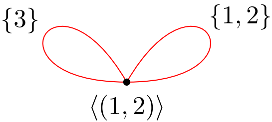

$\mathrm{Aut}(T_3)_\omega$

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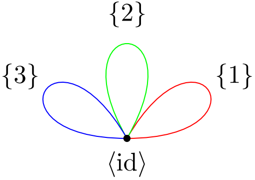

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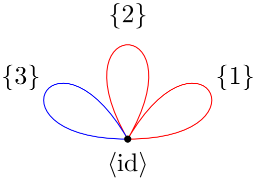

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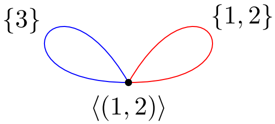

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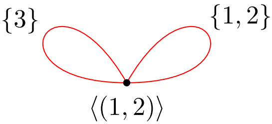

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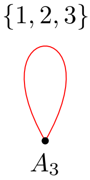

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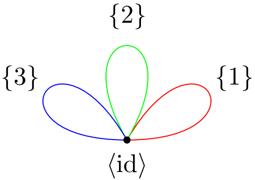

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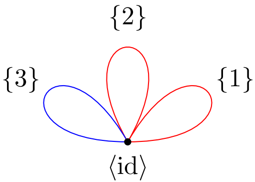

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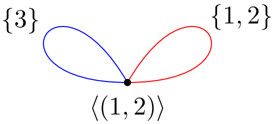

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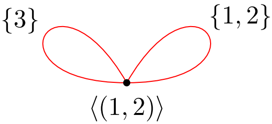

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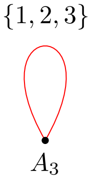

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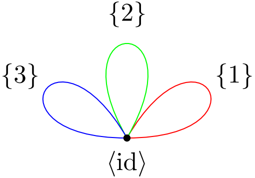

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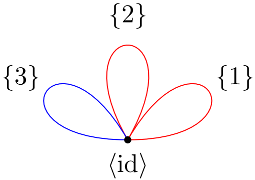

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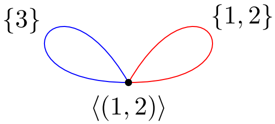

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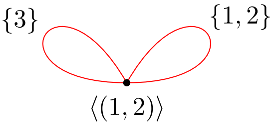

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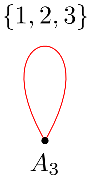

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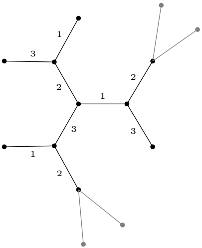
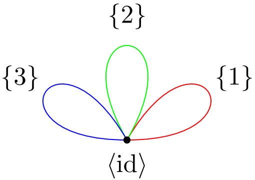


Example: vertex-transitive, (P_1)-closed $G \leq \mathrm{Aut}(T_3)$



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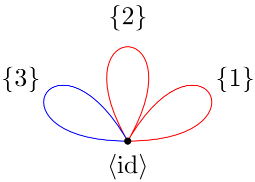

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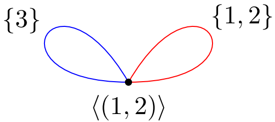

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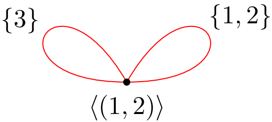

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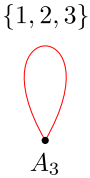

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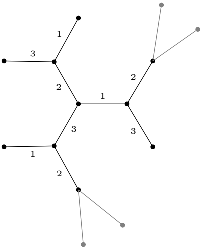

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Towards a classification of closed vertex-transitive groups