Introduction (Thin groups, IMS, 14/06/24, 60 minutes, joint with Marcus Chijoff)

- oddly specific?

Why groups acting on trees?

- groups acting on trees, buildings; p-adic matrix groups, Kac-Moody groups
- vertices of Cayley-Abels graph or cosets of a chosen compact open subgroup (can be arbitrarily small)

(P_k)-closed groups

- $(P_1) = (P)$ Tits' independence property
- every discrete group has (P_k) for some k (cf. Jeroen's talk)

Towards a classification of ...

- Example:

0 $\quad \mathrm{Aut}(T_3)$

1 $\quad \mathrm{Aut}(T_3) = U_1(S_3)$

2 $\quad \mathrm{Aut}(T_3) = U_2(\Phi(S_3))$

3 $\quad U_3(\ldots) \quad (384 < 3072) \quad \ldots$

4 $\quad \ldots \quad \ldots \quad \ldots$

$\vdots \quad \vdots$

end: $\quad PGL(2,\mathbb{Q}_2) \curvearrowright T_3 \quad \mathrm{Aut}(T_3)$

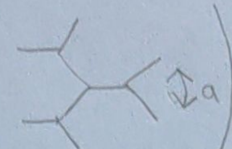
not (P_k)-closed for any k

$PGL(2,\mathbb{Q}_p)_v = PGL(2,\mathbb{Z}_p)$

$(PGL(2,\mathbb{Z}_p) \curvearrowright S(v,n)) \cong (PGL(2,\mathbb{Z}_p) \curvearrowright P^1(\mathbb{Z}_p/p^n\mathbb{Z}_p))$

Universal Groups

- resembles definition of smooth map between manifolds ?
- note: not a one-to-one correspondence between groups F and groups $U_k(F)$:

$U_2\Big($  $\Big) = U_2(\{id, a\}) = U_2(\{id\})$

$$U_k(F) = U_k(C(F))$$

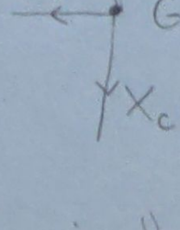
$C(F) \leq F$ unique subgroup that is maximal w.r.t. satisfying a compatibility condition

- these groups are (P_k)-closed ⇝ PhD thesis, GAP package

Classification results / plans

- boundary 2-transitive ⟹ locally 2-transitive (symmetric & alternating simplest)

Local action diagrams

- develop tree from diagram by hand
- 

$G \leq Sym(X_a) \times Sym(X_b) \times Sym(X_c)$

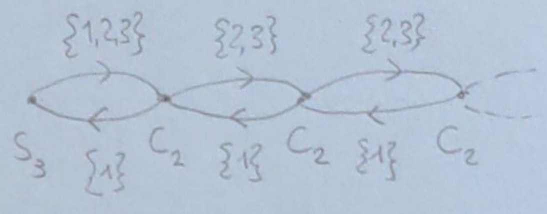
- group is the universal group for action on outgoing arcs

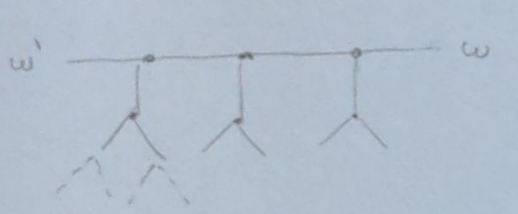
$$U(\Delta) := \{ g \in Aut_{\pi}(T) \mid \forall v \in V(T) : \sigma(g, v) \in G(\pi(v)) \leq Sym(X_{\pi(v)}) \}$$

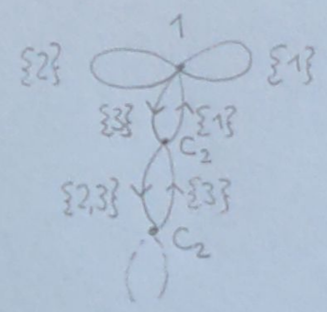
Correspondence

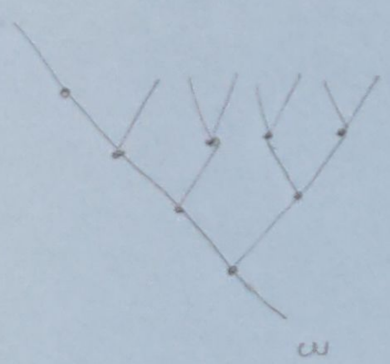
Six types of groups acting on trees

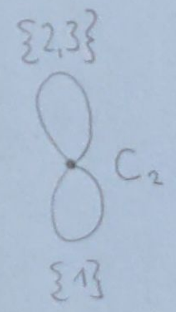
- first example that comes to mind is (P_1)-closed
- focus on fixed vertex, lineal and focal











Discrete (P)-closed groups

- explain theorem with three diagrams above